INTRODUCTION TO FINITE ELEMENT ANALYSIS OF COMPOSITE STRUCTURES

CO-HOSTED BY:

CKN
Composites Knowledge Network

sampe
Canada

compositeskn.org

nasampe.org

YOUR HOSTS

Alastair Komus, M.Sc., P.Eng.
Partner, A&S Composites Engineering

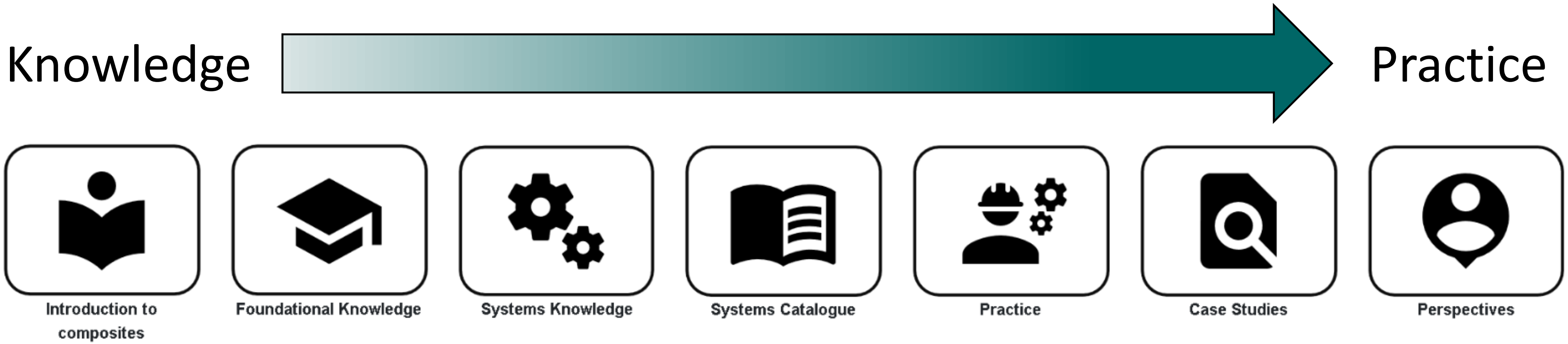
- 20 years of experience in design, analysis, testing, and manufacturing of composite structures
- Industries supported include aerospace, defence, marine, mass transit, agriculture, industrial, energy, sporting goods, and thrill rides
- Registered Professional Engineer from Ontario through BC
- Responsible for ensuring engineering and manufacturing capabilities at A&S Composites Engineering seamlessly integrate through all stages of product development

KNOWLEDGE IN PRACTICE CENTRE (KPC)

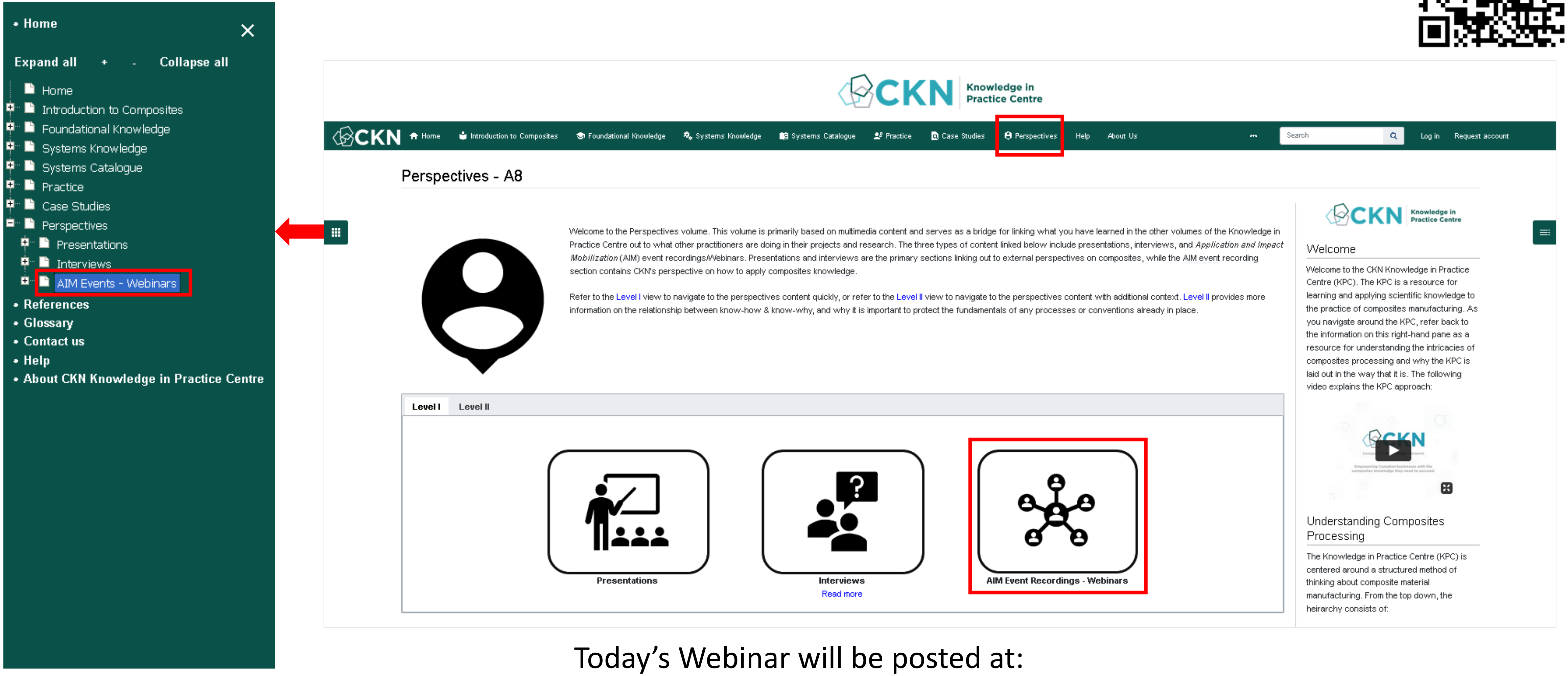
- A freely available online resource for composite materials engineering:

compositeskn.org/KPC

- Focus on practice, guided by foundational knowledge and a systems-based approach to thinking about composites manufacturing



PAST WEBINAR RECORDINGS AVAILABLE



Today's Webinar will be posted at:
https://compositeskn.org/KPC/A392

HOSTED BY: CKN Composites Knowledge Network

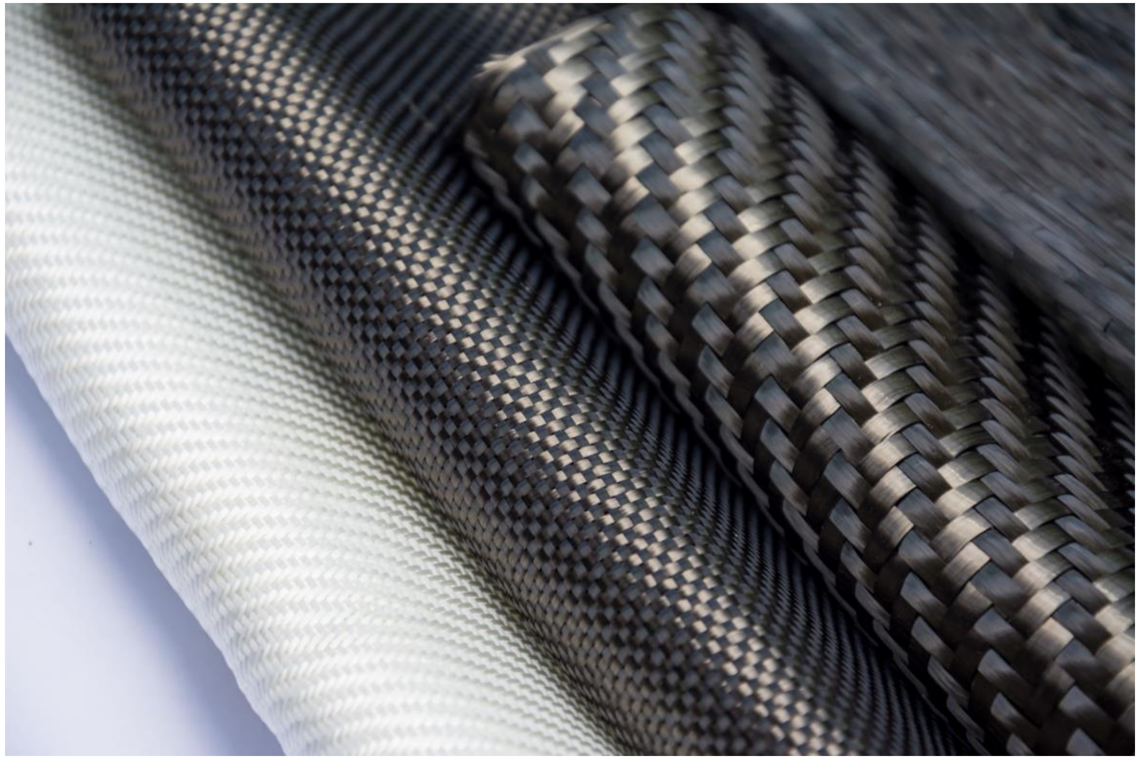
HOSTED BY: sampe Canada

TODAY'S TOPIC:

Introduction To Finite Element Analysis Of Composite Structures

OUTLINE

- Composite vs. metallic analysis
- Material properties and laminate definition
- Types of elements
- Ply orientation and draping
- Failure theories
- Interpreting results

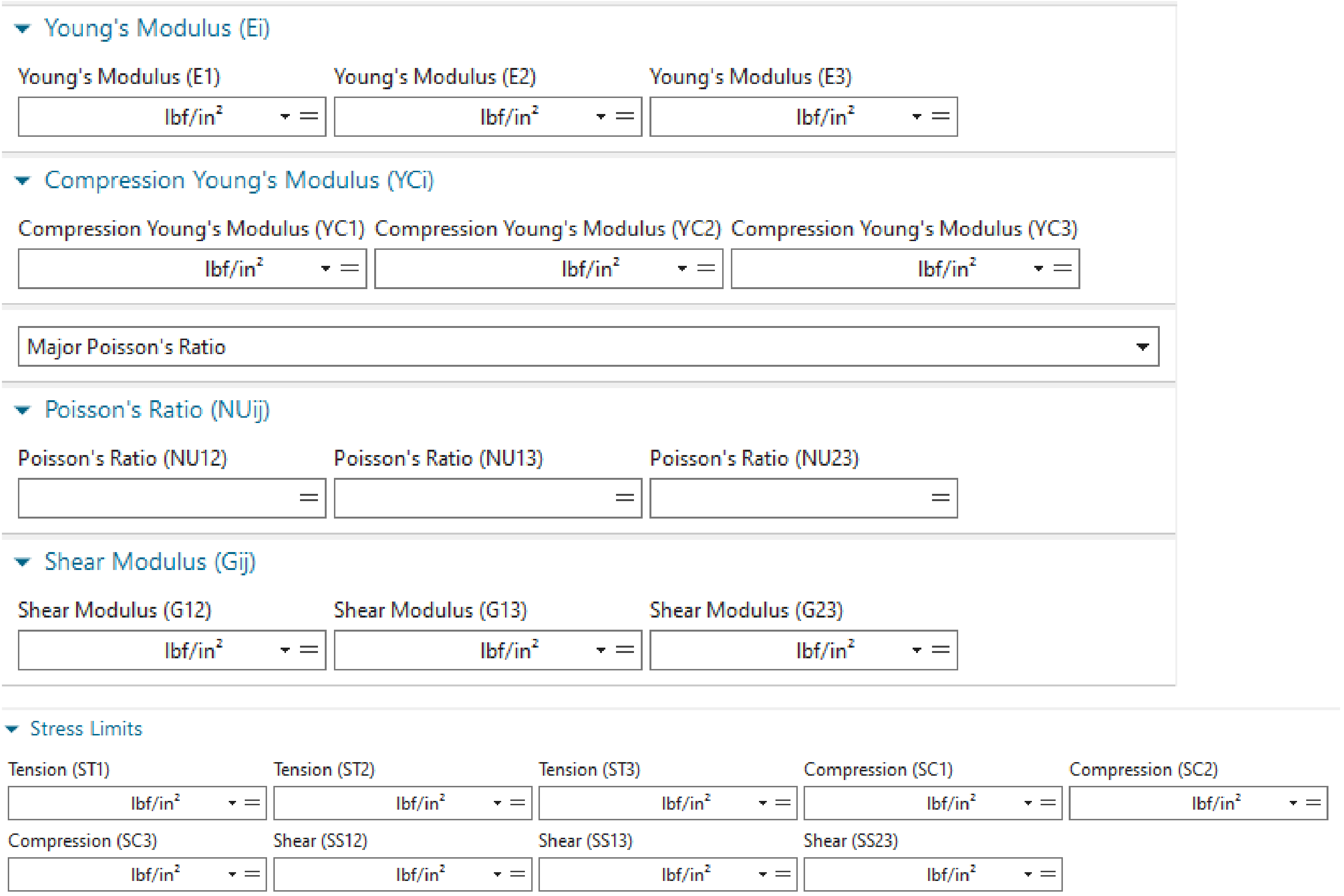


COMPOSITES VS. METALLICS ANALYSIS

- Isotropic
  - Material properties are the same in all directions
  - e.g. steel and aluminum
- Orthotropic
  - Material properties vary along 3 mutually perpendicular directions
  - e.g. carbon fibre epoxy prepreg
- Layup definition and ply orientation are critical parameters
- Ply patches and drop offs need to be considered
- Through thickness stresses are often ignored for composite analysis or require special consideration
- Different failure mechanisms

MATERIAL PROPERTIES

- Understanding of the manufacturing method required to estimate fibre content
- Need to define
  - Longitudinal modulus
  - Transverse modulus
  - Shear modulus
  - Poisson's ratio
  - Longitudinal tensile strength
  - Transverse tensile strength
  - Longitudinal compressive strength
  - Transverse compressive strength
  - Shear strength
  - Ply thickness



MATERIAL PROPERTIES

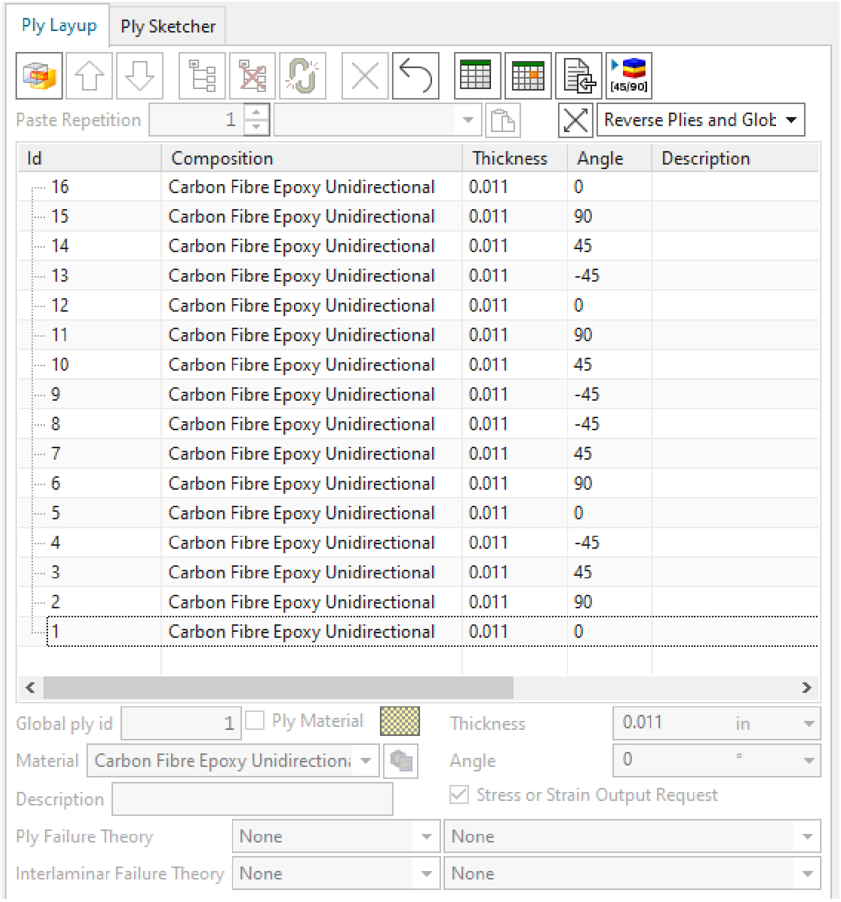
- Potential material property sources
  - Micromechanics (usually only for preliminary studies)
  - Supplier datasheets (use with caution)
  - Trusted databases (e.g. AGATE)
  - Internal test programs

$$E_1 = E_f V_f + E_m V_m$$

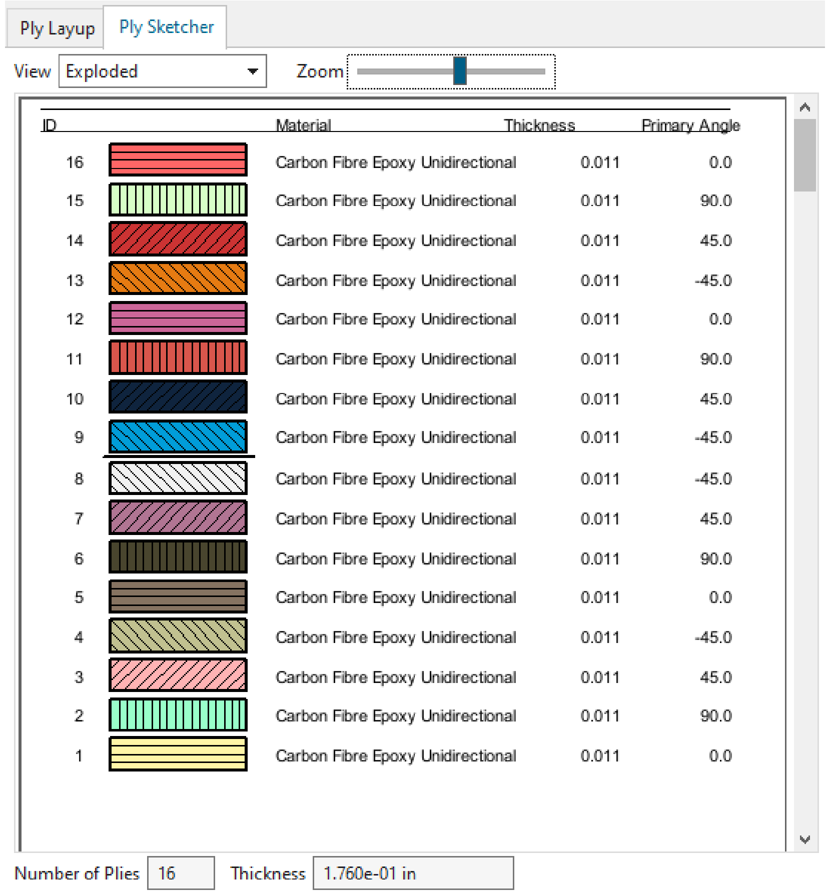
$$\frac{1}{E_2} = \frac{1}{E_f} V_f + \frac{1}{E_m} V_m$$

LAMINATE DEFINITION

- Ply by ply definition including:
  - Material type
  - Thickness
  - Orientation



Id	Composition	Thickness	Angle	Description
16	Carbon Fibre Epoxy Unidirectional	0.011	0	
15	Carbon Fibre Epoxy Unidirectional	0.011	90	
14	Carbon Fibre Epoxy Unidirectional	0.011	45	
13	Carbon Fibre Epoxy Unidirectional	0.011	-45	
12	Carbon Fibre Epoxy Unidirectional	0.011	0	
11	Carbon Fibre Epoxy Unidirectional	0.011	90	
10	Carbon Fibre Epoxy Unidirectional	0.011	45	
9	Carbon Fibre Epoxy Unidirectional	0.011	-45	
8	Carbon Fibre Epoxy Unidirectional	0.011	-45	
7	Carbon Fibre Epoxy Unidirectional	0.011	45	
6	Carbon Fibre Epoxy Unidirectional	0.011	90	
5	Carbon Fibre Epoxy Unidirectional	0.011	0	
4	Carbon Fibre Epoxy Unidirectional	0.011	-45	
3	Carbon Fibre Epoxy Unidirectional	0.011	45	
2	Carbon Fibre Epoxy Unidirectional	0.011	90	
1	Carbon Fibre Epoxy Unidirectional	0.011	0	



ID	Material	Thickness	Primary Angle
16	Carbon Fibre Epoxy Unidirectional	0.011	0.0
15	Carbon Fibre Epoxy Unidirectional	0.011	90.0
14	Carbon Fibre Epoxy Unidirectional	0.011	45.0
13	Carbon Fibre Epoxy Unidirectional	0.011	-45.0
12	Carbon Fibre Epoxy Unidirectional	0.011	0.0
11	Carbon Fibre Epoxy Unidirectional	0.011	90.0
10	Carbon Fibre Epoxy Unidirectional	0.011	45.0
9	Carbon Fibre Epoxy Unidirectional	0.011	-45.0
8	Carbon Fibre Epoxy Unidirectional	0.011	-45.0
7	Carbon Fibre Epoxy Unidirectional	0.011	45.0
6	Carbon Fibre Epoxy Unidirectional	0.011	90.0
5	Carbon Fibre Epoxy Unidirectional	0.011	0.0
4	Carbon Fibre Epoxy Unidirectional	0.011	-45.0
3	Carbon Fibre Epoxy Unidirectional	0.011	45.0
2	Carbon Fibre Epoxy Unidirectional	0.011	90.0
1	Carbon Fibre Epoxy Unidirectional	0.011	0.0

LAMINATE DEFINITION

- Software calculates the stiffness matrices using the entered data
- Important to be aware of possible coupling effects

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{21} & A_{22} & A_{26} & B_{21} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{21} & B_{22} & B_{26} & D_{21} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \\ \kappa_x \\ \kappa_y \\ \kappa_z \end{Bmatrix}$$

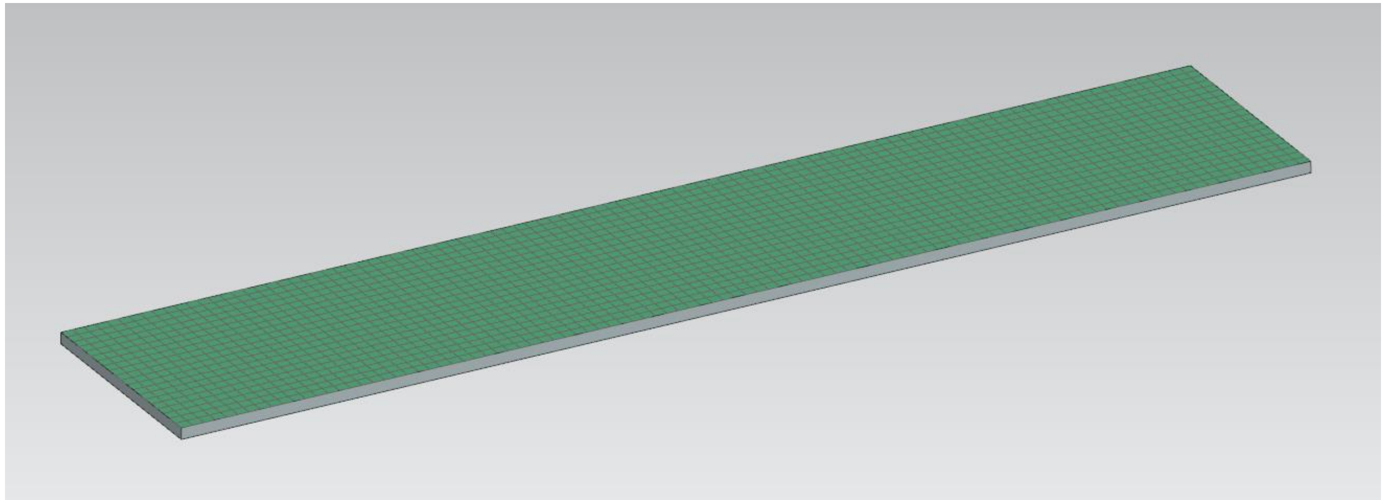
```
In-Plane Stiffness Matrix A          : |   1.268e+06  4.040e+05  0.000e+00|
                                       |   4.040e+05  1.268e+06  0.000e+00|
                                       |   0.000e+00  0.000e+00  4.318e+05|
                                       lbf/in

Coupling Stiffness Matrix B          : |  -7.297e+03  3.607e+03 -1.845e+03|
                                       |   3.607e+03  8.284e+01 -1.845e+03|
                                       |  -1.845e+03 -1.845e+03  3.607e+03|
                                       lbf

Flexural Stiffness Matrix D          : |   3.353e+03  1.043e+03 -4.059e+01|
                                       |   1.043e+03  3.191e+03 -4.059e+01|
                                       |  -4.059e+01 -4.059e+01  1.115e+03|
                                       lbf-in
```

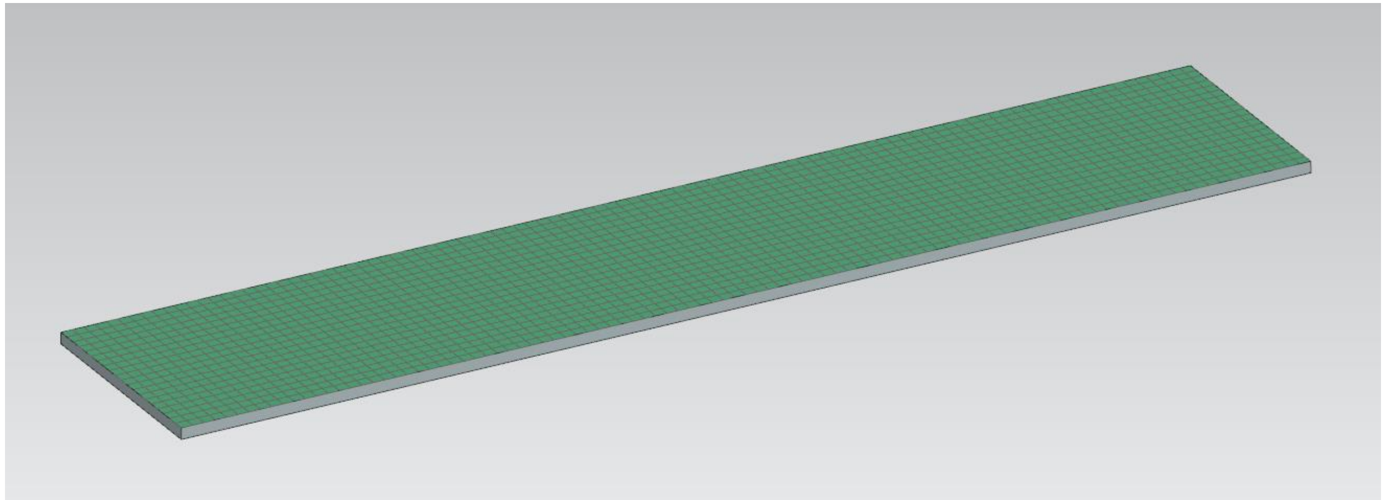
TYPES OF ELEMENTS

- 2D shell elements are the most common for analysis of composite structures
- Each solver has their own name for the element type
- Assumes plane stress condition
- Most solvers use First-order Shear Deformation Theory (FSDT) for analysis
  - Similar to Classical Lamination Theory (CLT) but CLT assumes no transverse shear deformation
  - FSDT assumes a constant shear deformation and force through the thickness of the laminate



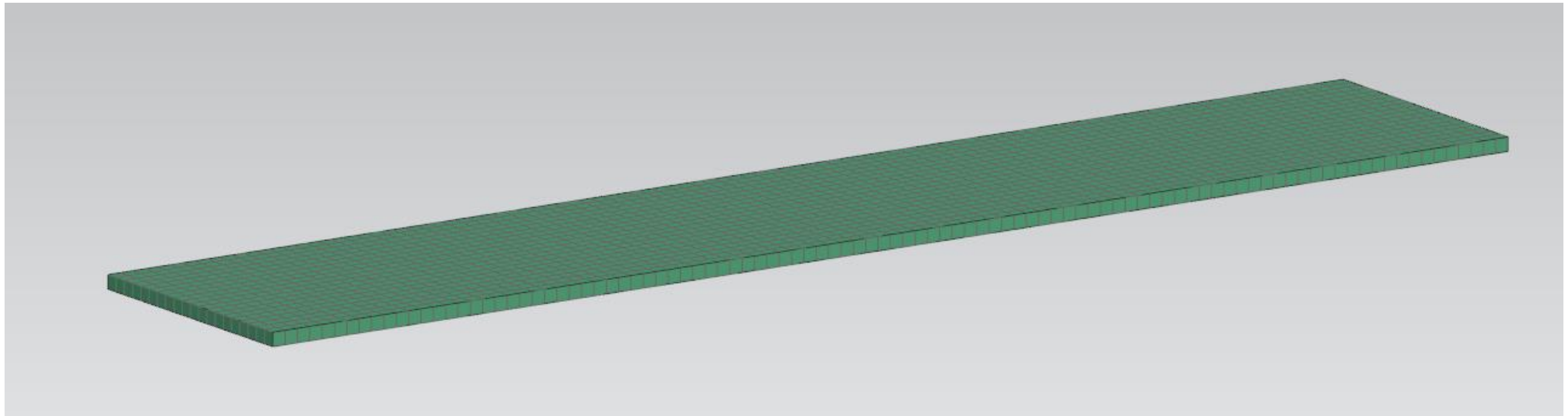
TYPES OF ELEMENTS

- 2D shell elements
  - Suitable for thin structures
  - Quick setup
  - Easy to change ply thickness
  - Through thickness stress is not calculated



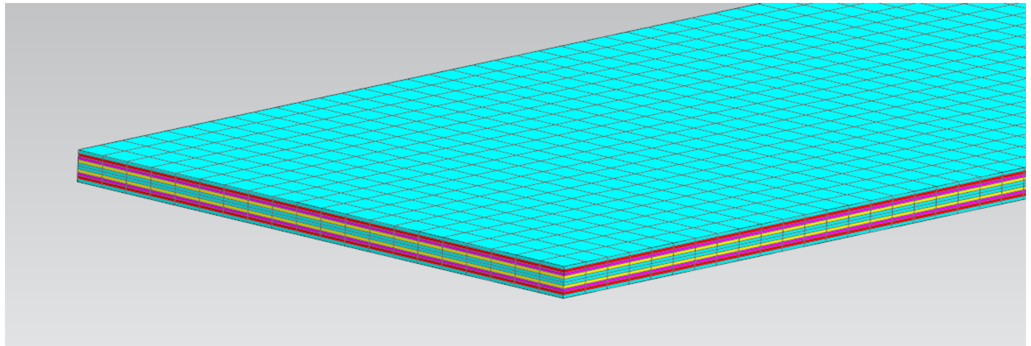
TYPES OF ELEMENTS

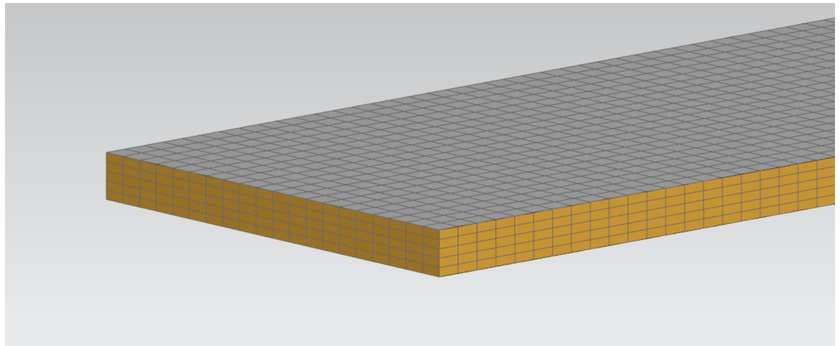
- 3D solid elements used when through thickness stress and interlaminar shear stress are critical
  - Joints, edges, cored structures
- 3D layered elements
  - Solid element type that defines the laminate layup
  - Careful when analysing bending as solid elements lack rotational degrees of freedom
  - Need to define through thickness material properties which may not be known



TYPES OF ELEMENTS

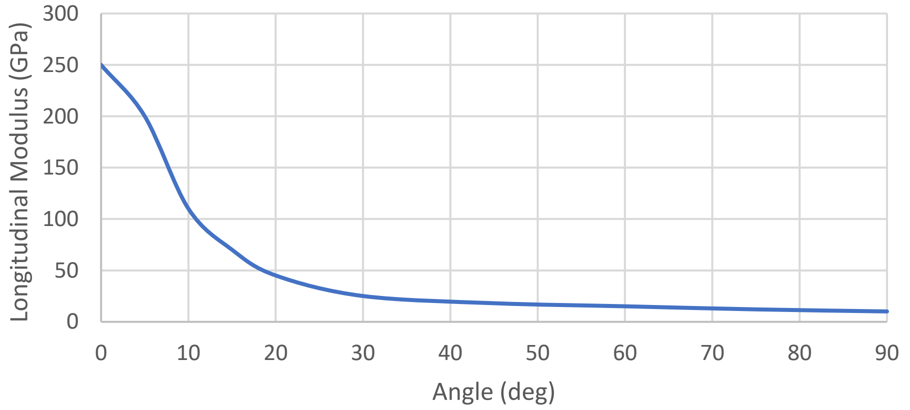
- 3D hexahedral or tetrahedral elements
  - Each ply is meshed as its own layer of elements
  - Core meshed using 3D elements and face sheets modelled using 2D shell elements
- Better able to capture through thickness stresses and deflections in the core
- Need to define through thickness material properties which may not be known
- More computationally intensive and longer setup time

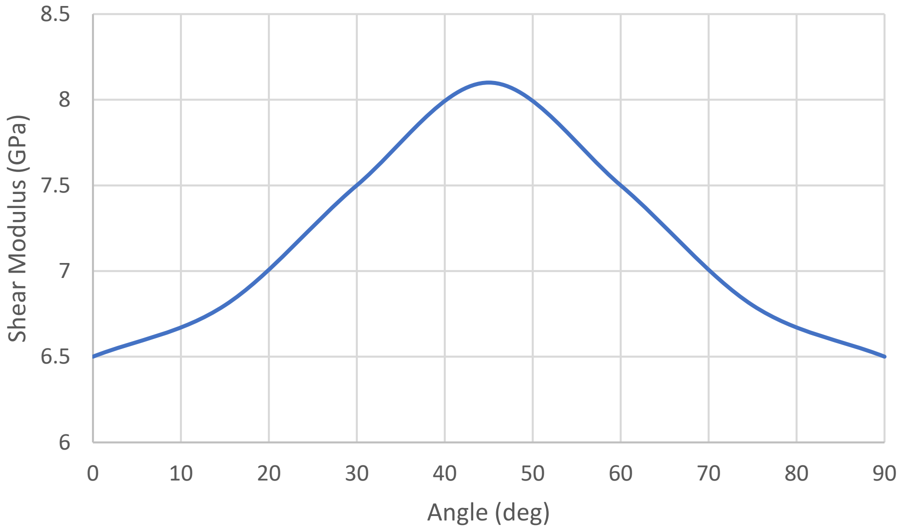




PLY ORIENTATION

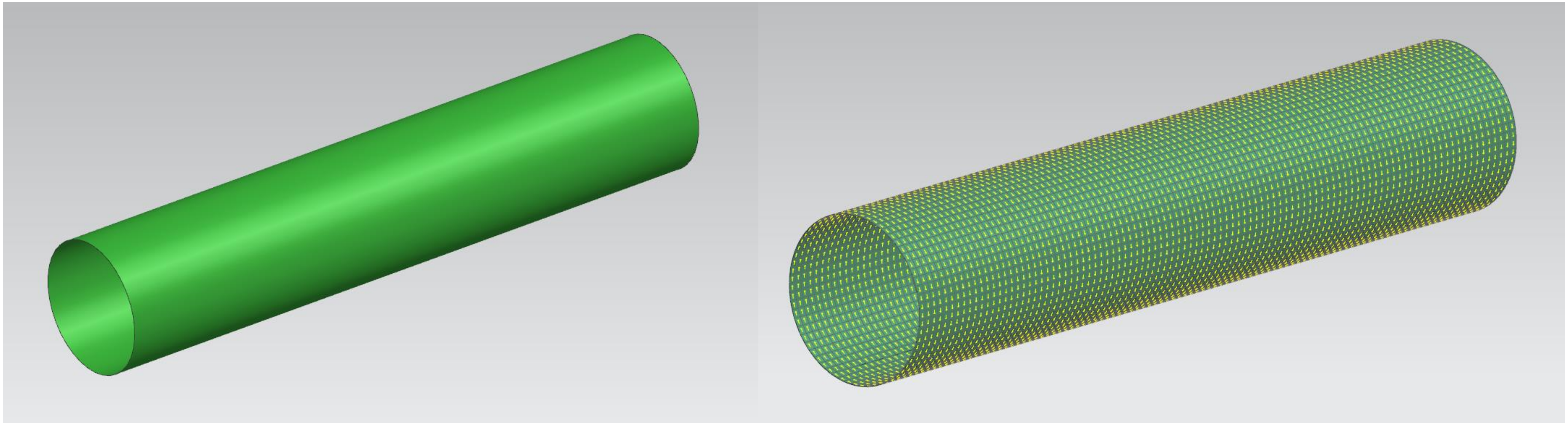
- Accurate ply orientation is critical to achieving meaningful results





PLY ORIENTATION

- Part geometry can dramatically affect the fibre orientation
- Fibre orientation of flat or singly curved surfaces remains practically constant and can be performed by defining material orientation manually

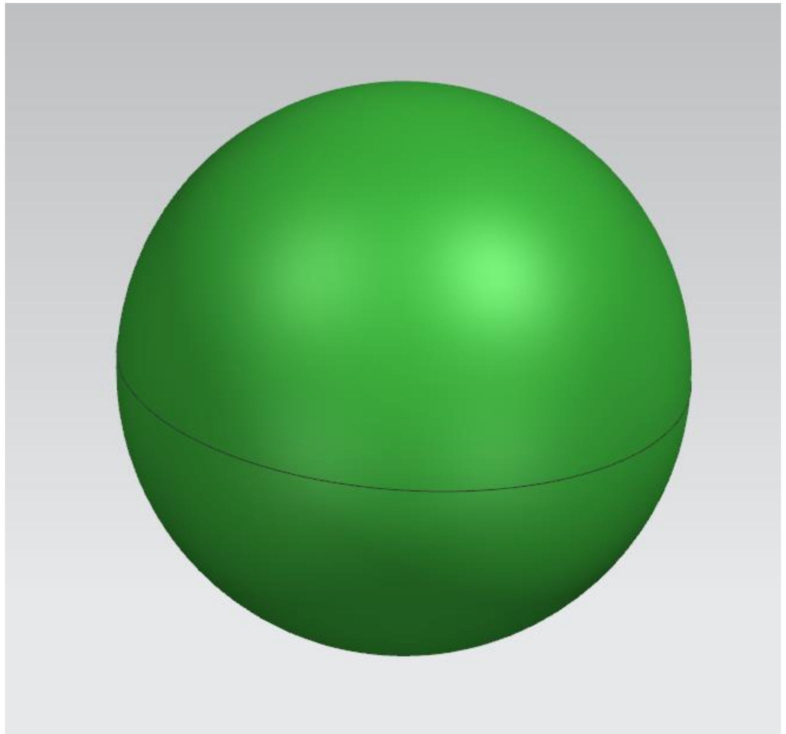


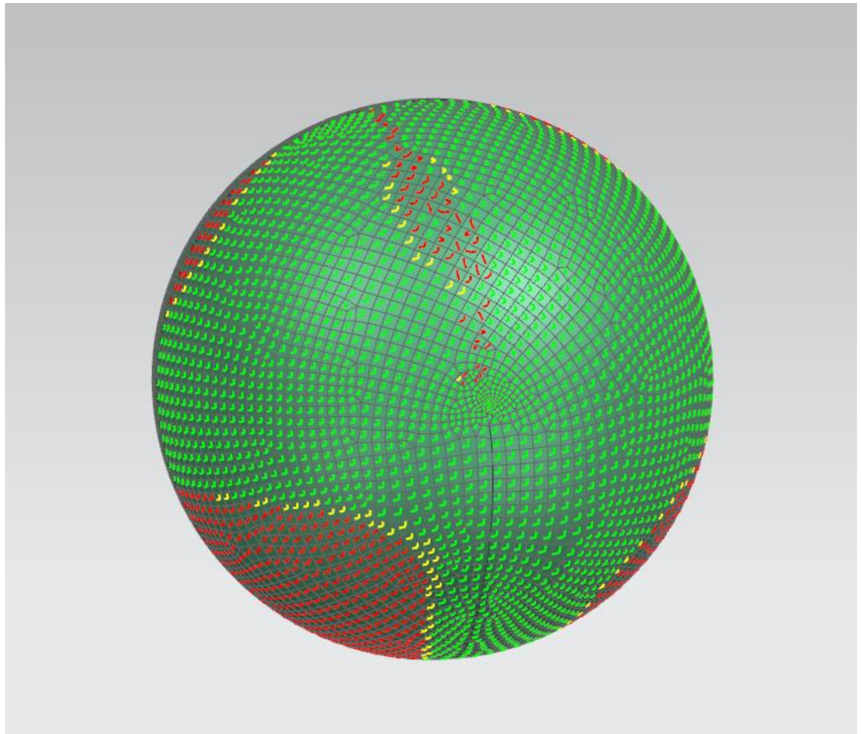
HOSTED BY: CKN Composites Knowledge Network

HOSTED BY: sampe Canada

PLY ORIENTATION

- For doubly curved surfaces the ply must deform to follow the surface
  - Deformation occurs by in-plane shearing
  - Fibre orientation changes



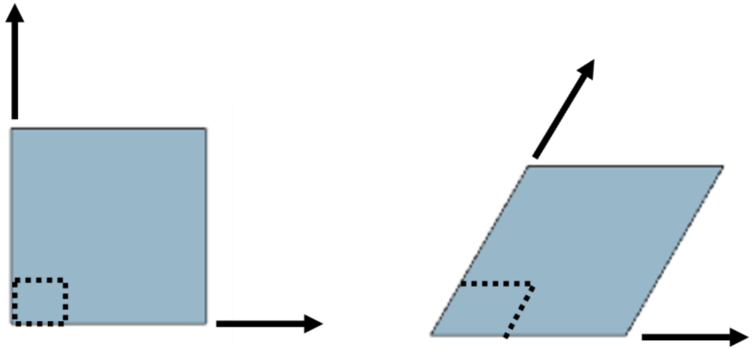


HOSTED BY: CKN Composites Knowledge Network

HOSTED BY: sampe Canada

DRAPING

- Ply draping simulation is helpful to:
  - Produce more accurate analysis results for doubly curved surfaces
  - Determine the location of splices or new plies
- Draping solvers vary depending on the software but most use similar methodologies
- Need to define a starting point and initial fibre alignment direction(s)
- Solver will accommodate changes in geometry by shear distortion



DRAPING

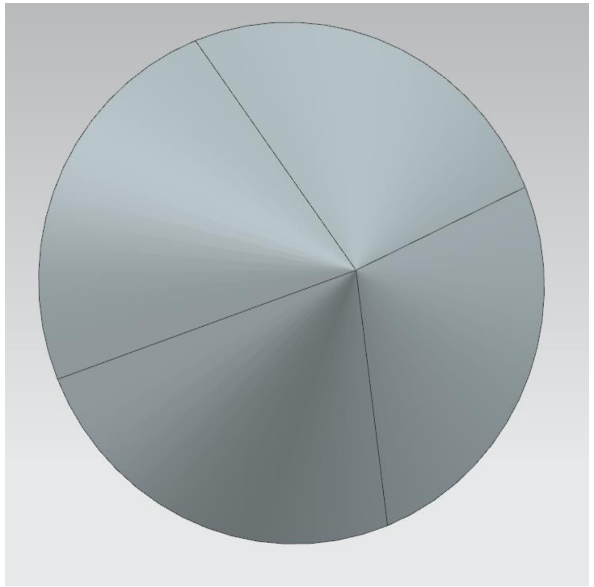
- If the shear angle becomes too large the ply begins to wrinkle or bridge
- Lock angle defines the maximum amount of in-plane shear deformation a fabric can withstand before beginning to deform out-of-plane
- Methods to resolve shearing problems include:
    - Reposition the draping start point
    - Adding cut curves to model darts in the ply

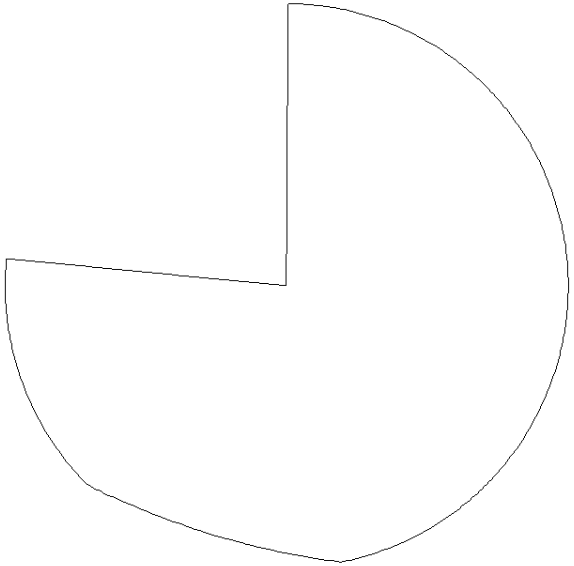
DRAPING

- Most woven fabric reinforcements have a lock angle between 30° to 60°
- Generally, stitched materials will have a smaller lock angle because the stitching inhibits in-plane shear deformation
- Standardized tests are available to quantify the fabric lock angle including
  - Picture frame
  - Bias extension
  - Biaxial shear

DRAPING

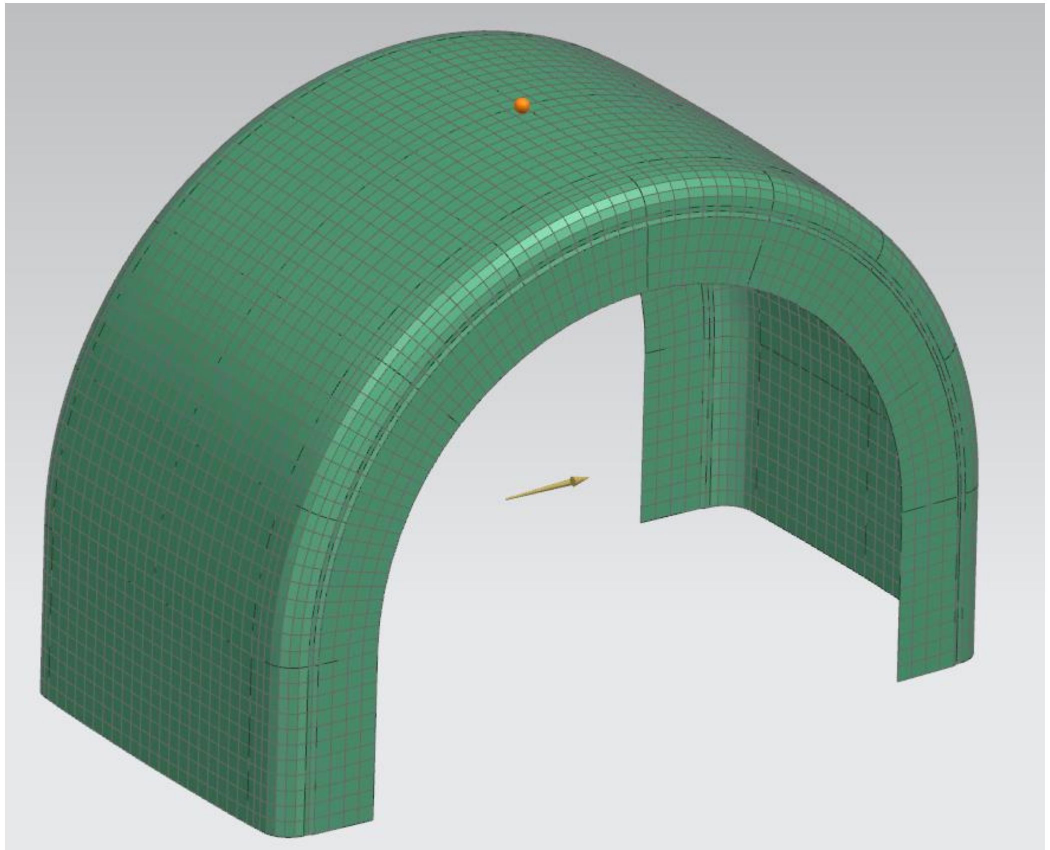
- Most draping software can produce a flat pattern based on the draping results and the specified darts
- Flat patterns can be used to create physical ply cutting templates
- Flat pattern data can be used to program automated ply cutting equipment
- Reduces ply cutting labour
- Increases accuracy and consistency results in improved part quality





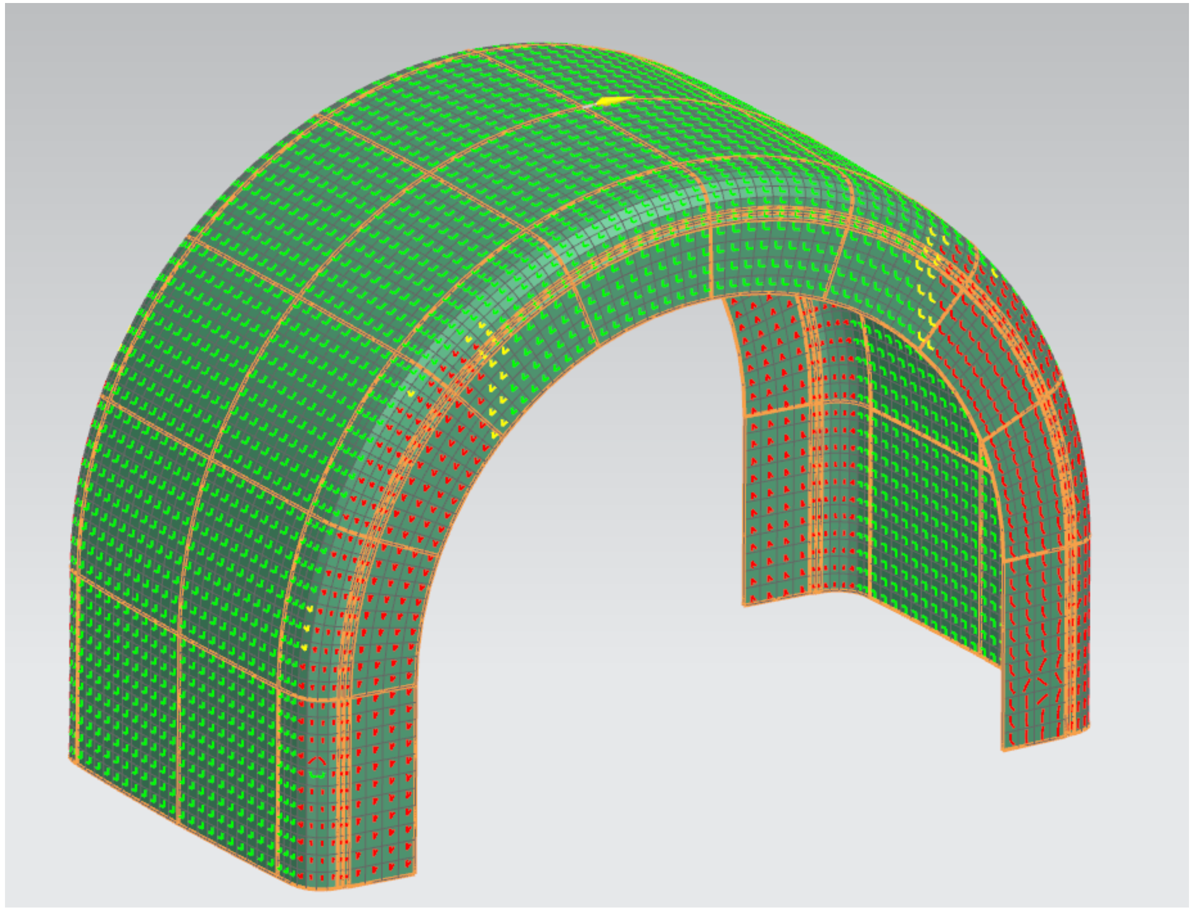
DRAPING – TRUCK FENDER EXAMPLE

- Truck Fender
  - Stitched 0/90 fabric
  - Start point at apex of the fender
  - Primary direction of fabric is longitudinal



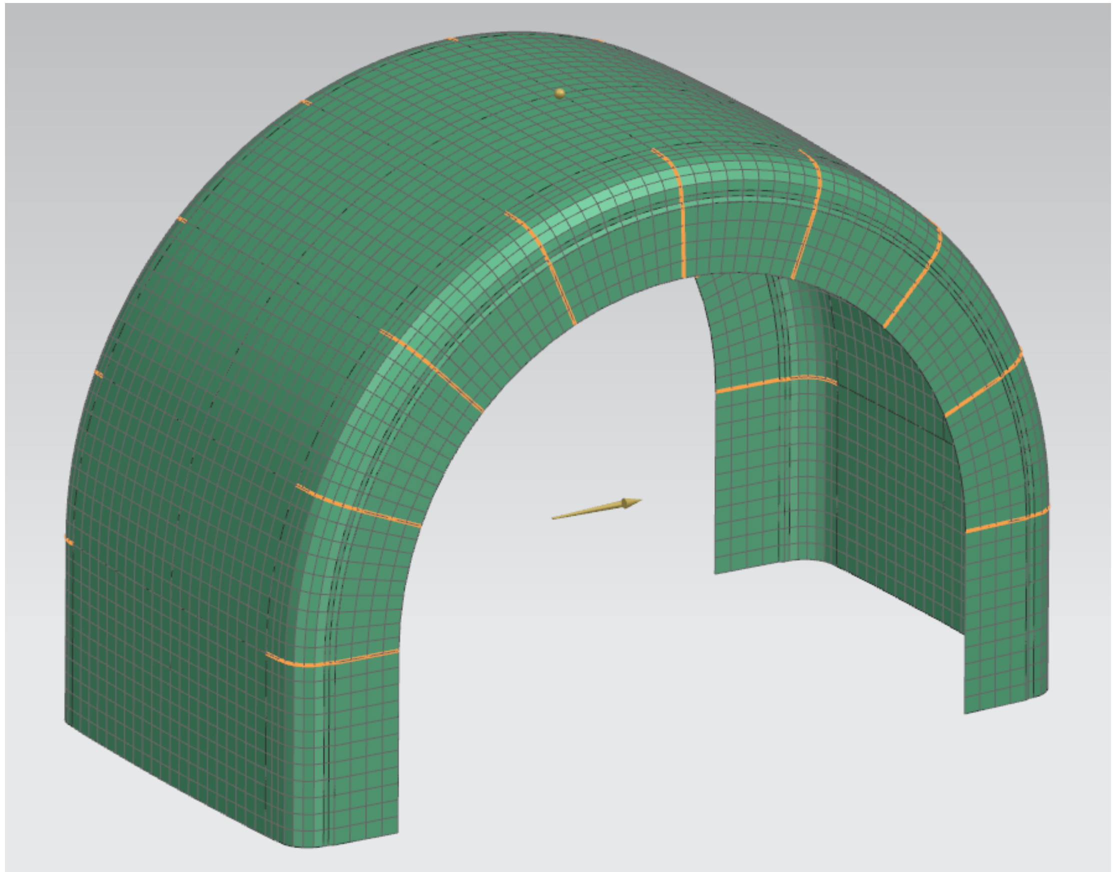
DRAPING – TRUCK FENDER EXAMPLE

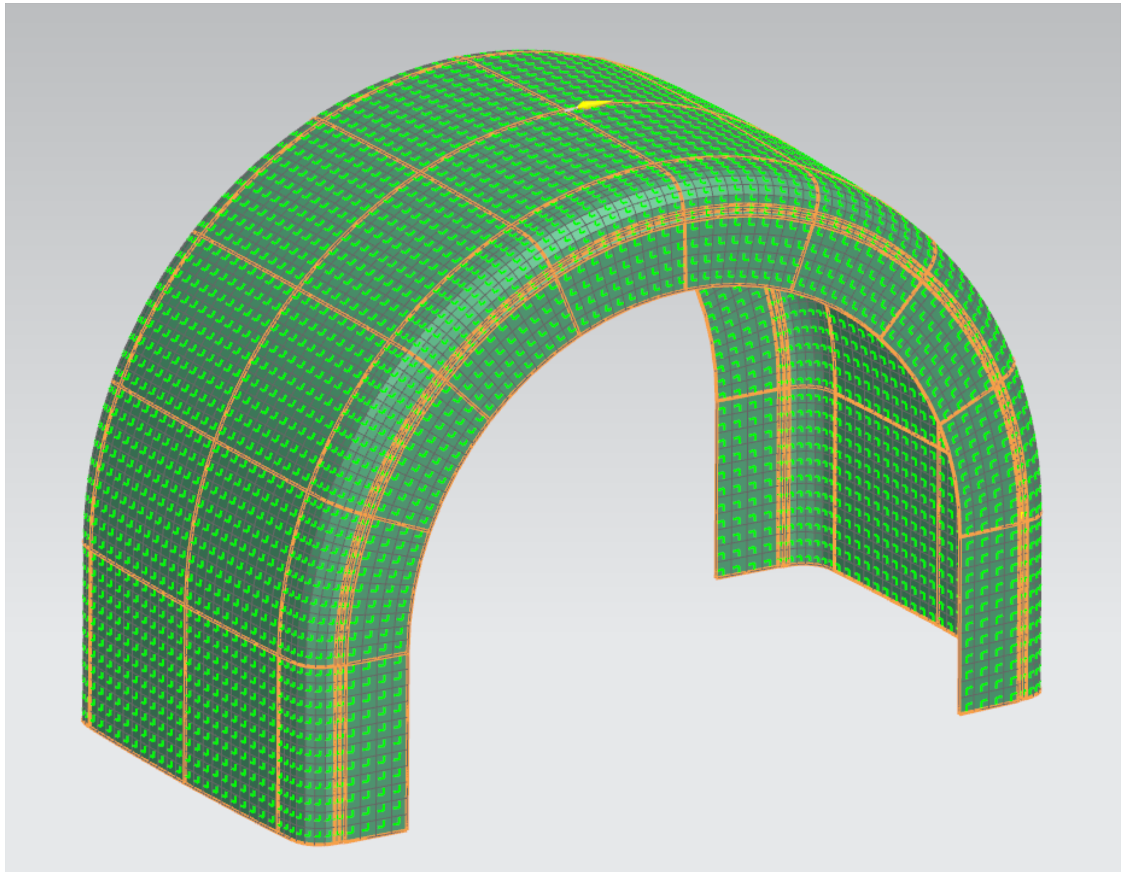
- Green – shear angle < 90% lock angle
- Yellow – 90% lock angle< shear angle< lock angle
- Red – shear angle > lock angle
- Result in wrinkling of the fabric



DRAPING – TRUCK FENDER EXAMPLE

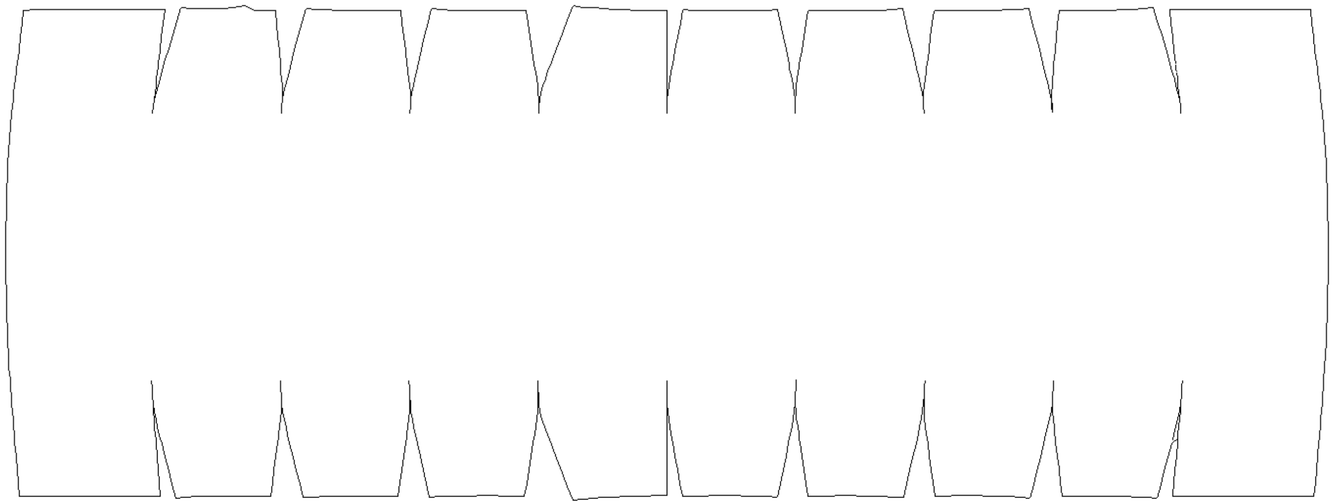
- Darts adding along the doubly curved surface





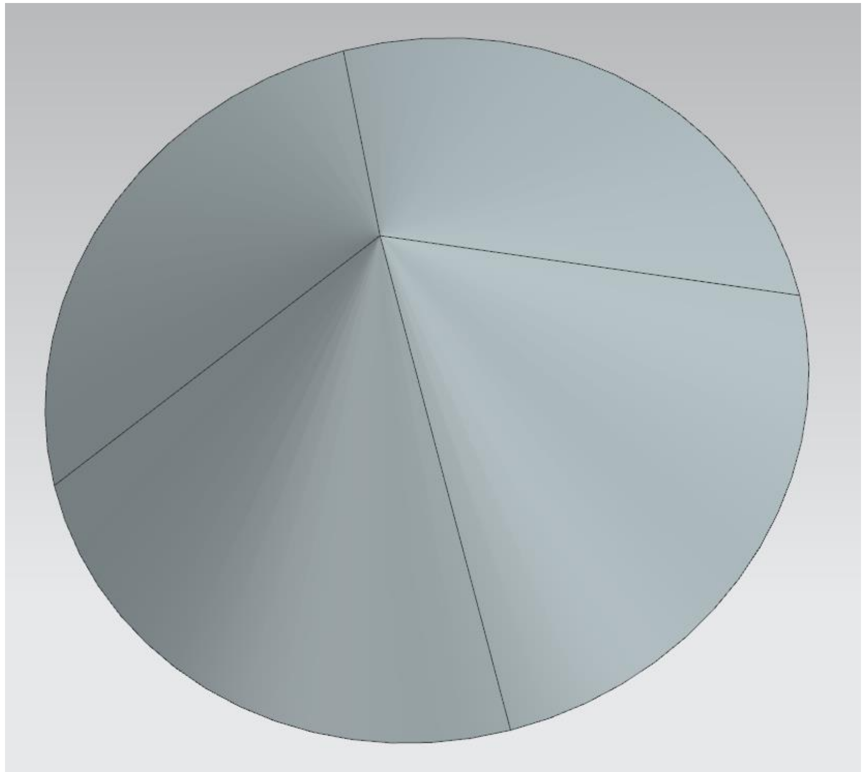
DRAPING – TRUCK FENDER EXAMPLE

- Flat Pattern
- Can be used to create template for cutting or exported to automated ply cutting software



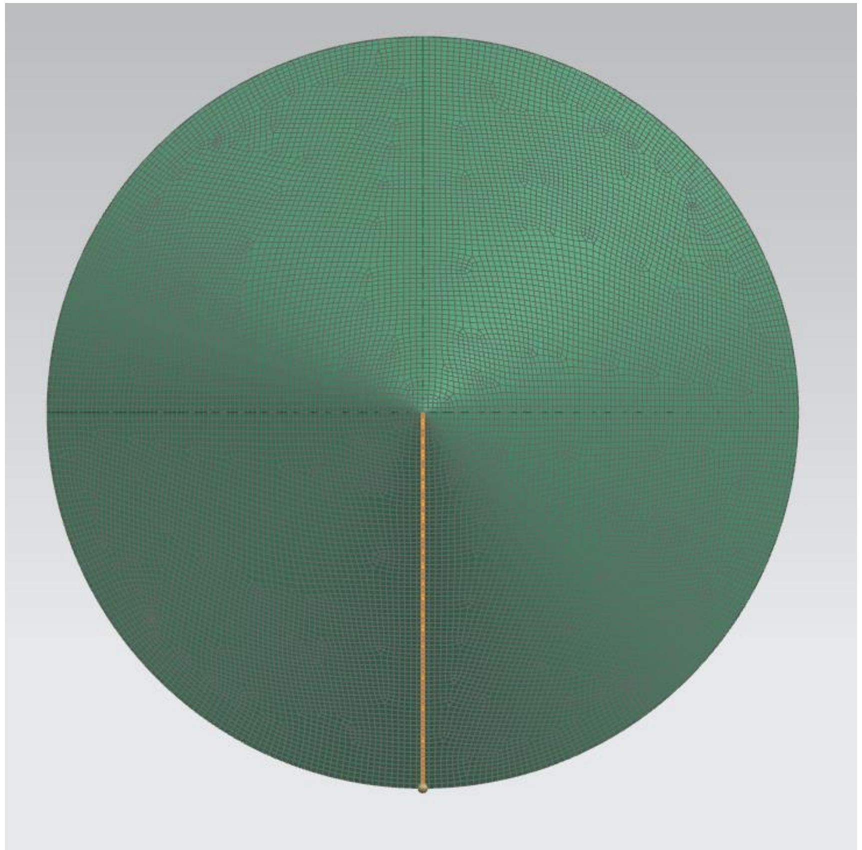
DRAPING – CONIC SHELL EXAMPLE

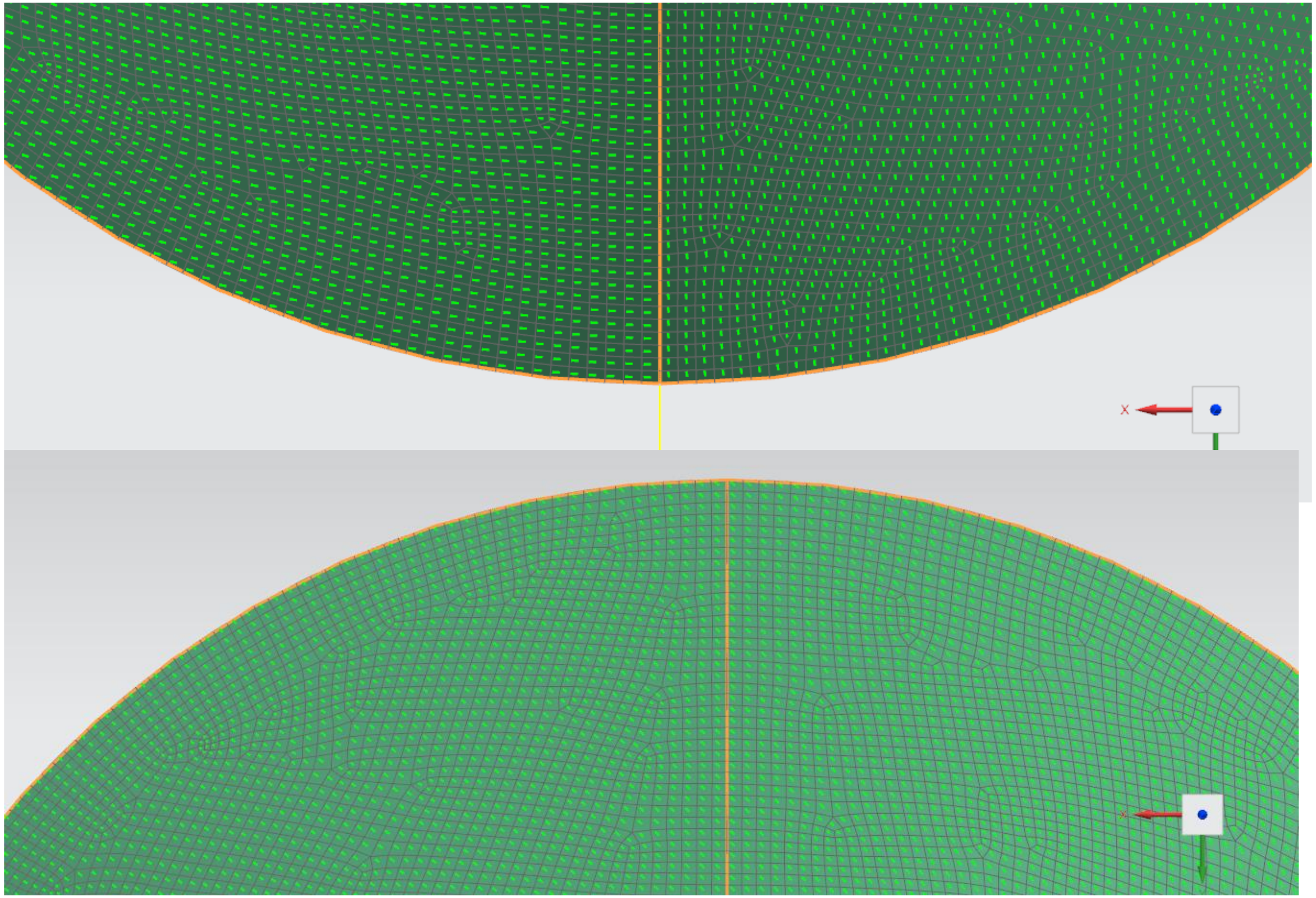
- Conic Shell
  - Using different draping strategies will result in different material orientations
  - Unidirectional fabric
  - Two strategies
    - Drape with one dart
    - Drape each quarter separately



DRAPING – CONIC SHELL EXAMPLE

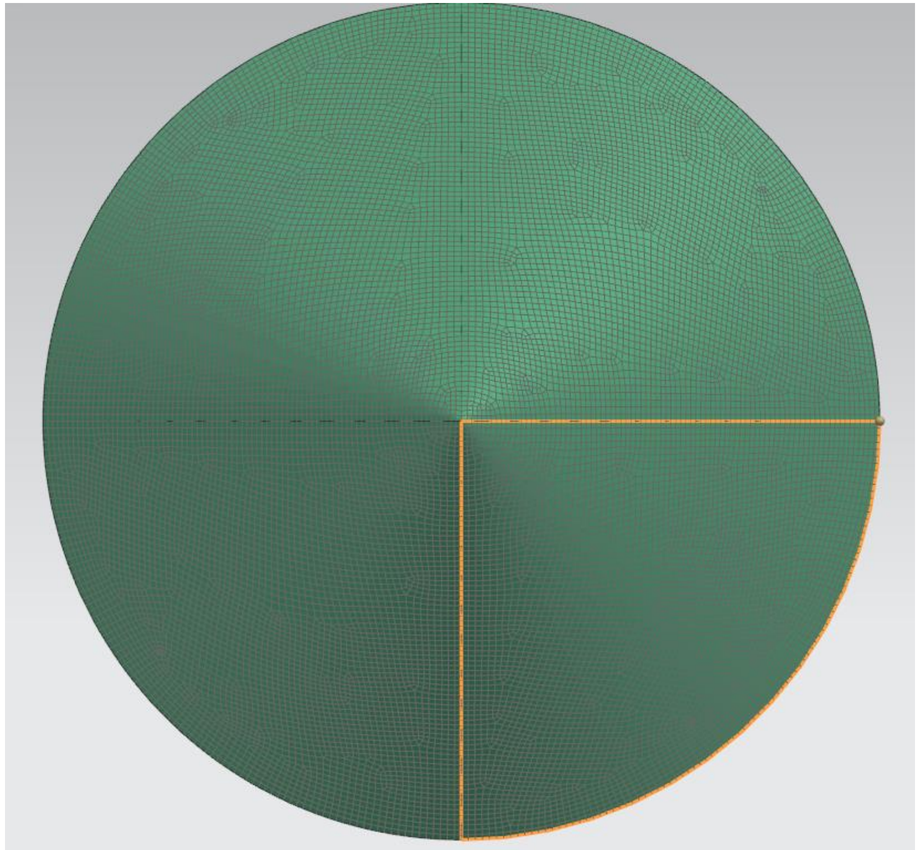
- Drape with one dart

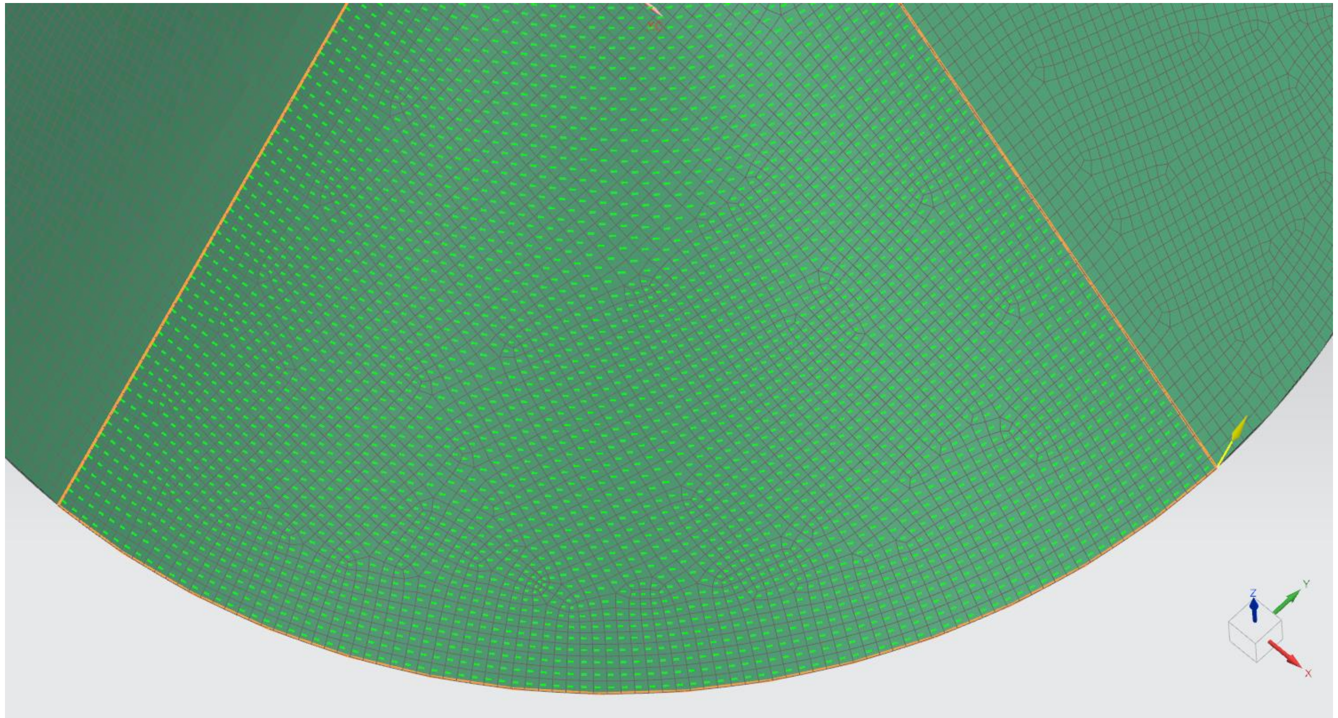




DRAPING – CONIC SHELL EXAMPLE

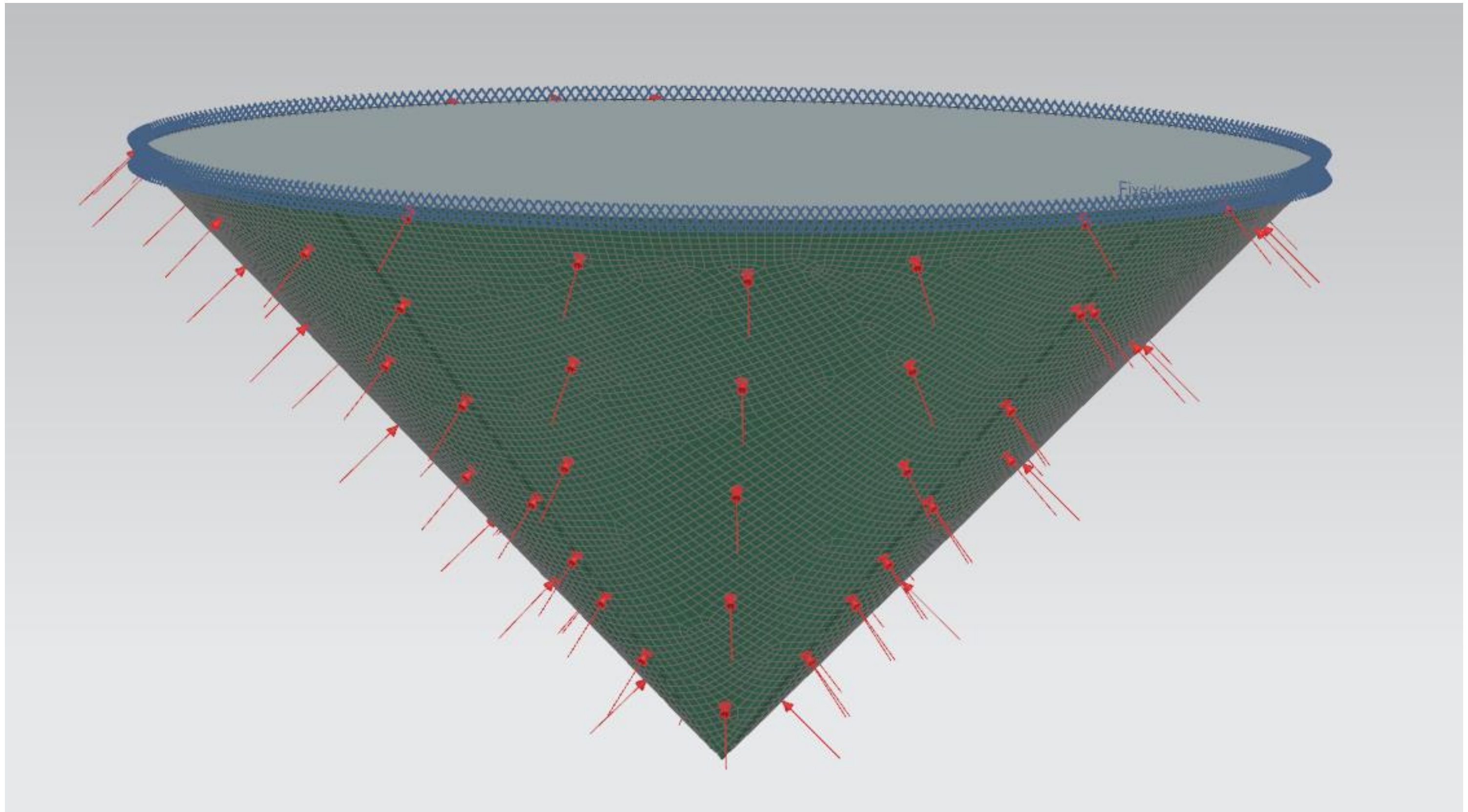
- Drape each quarter separately





DRAPING – CONIC SHELL EXAMPLE

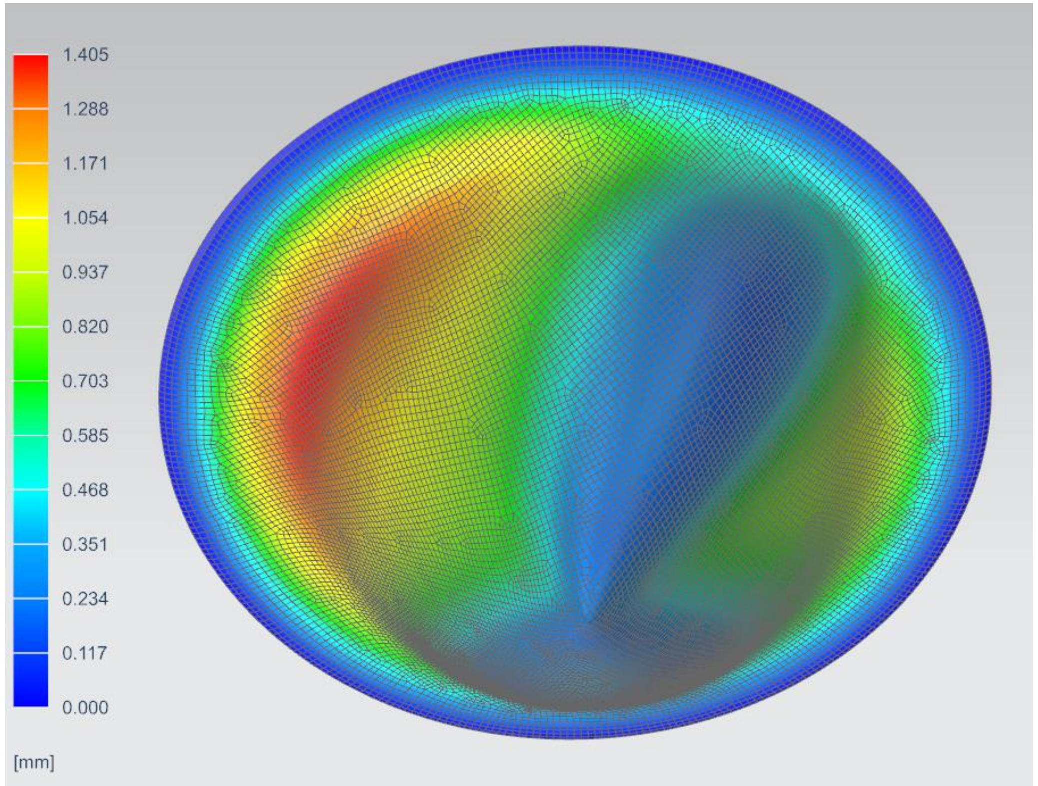
- Fixed at top edge
- Pressure applied to conical faces



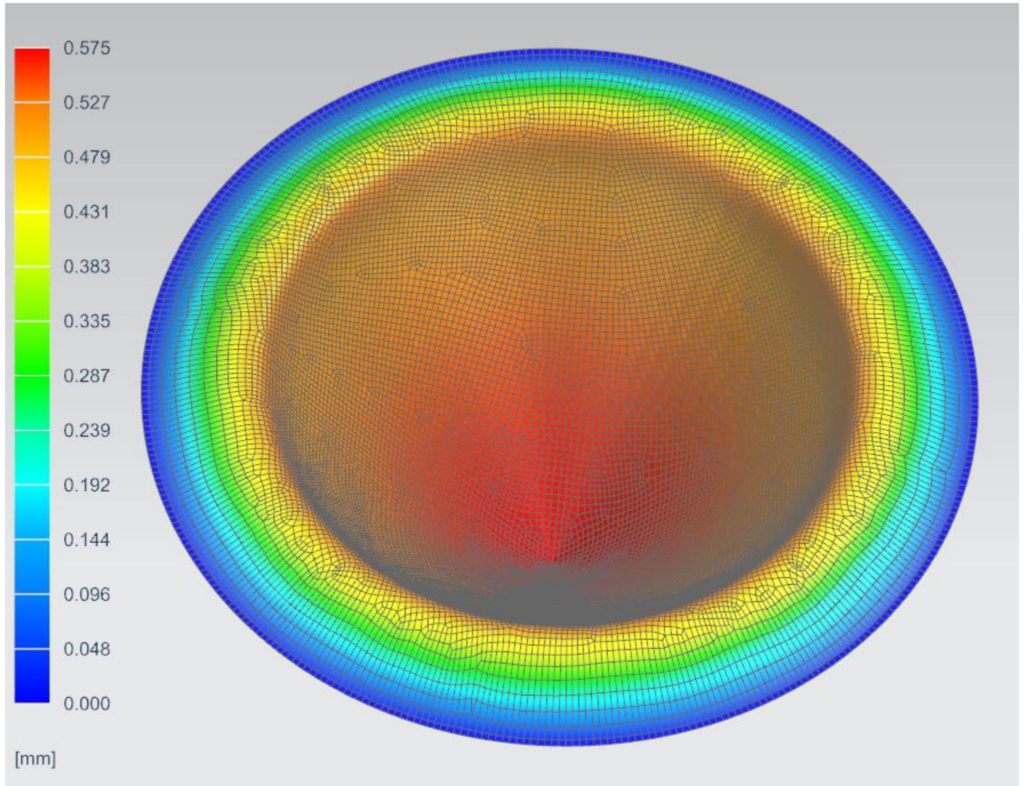
DRAPING – CONIC SHELL EXAMPLE

- Deflection

One Dart



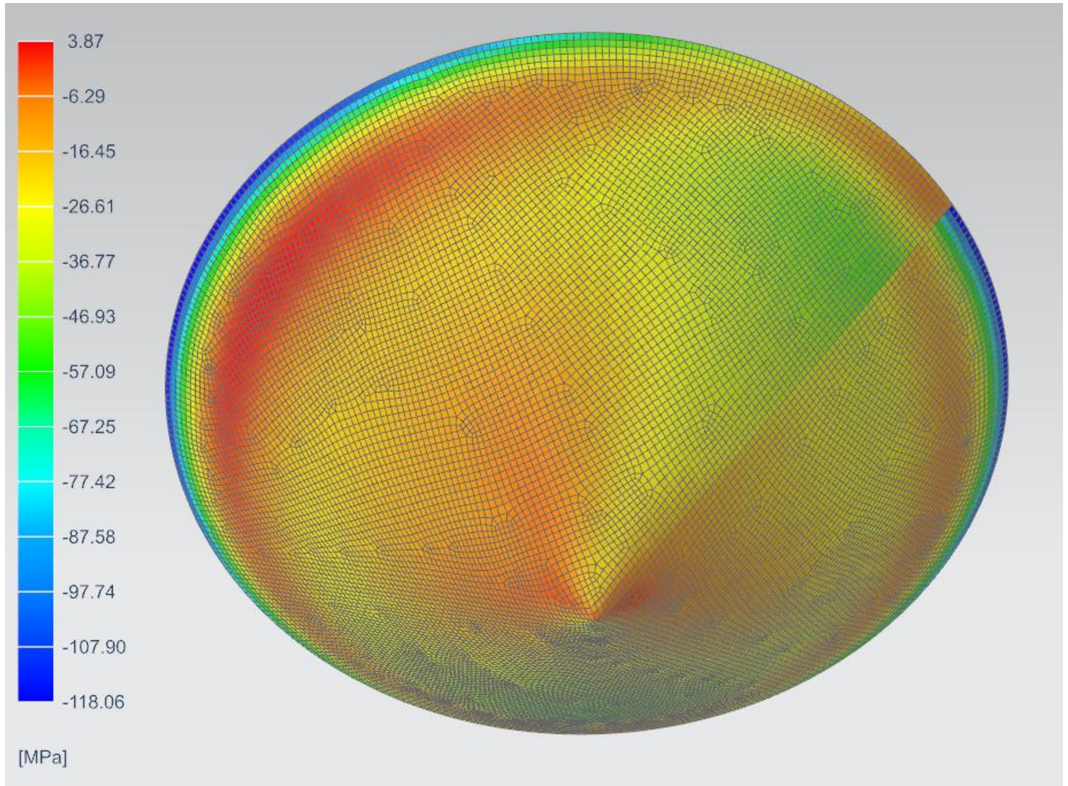
Separate Quarters



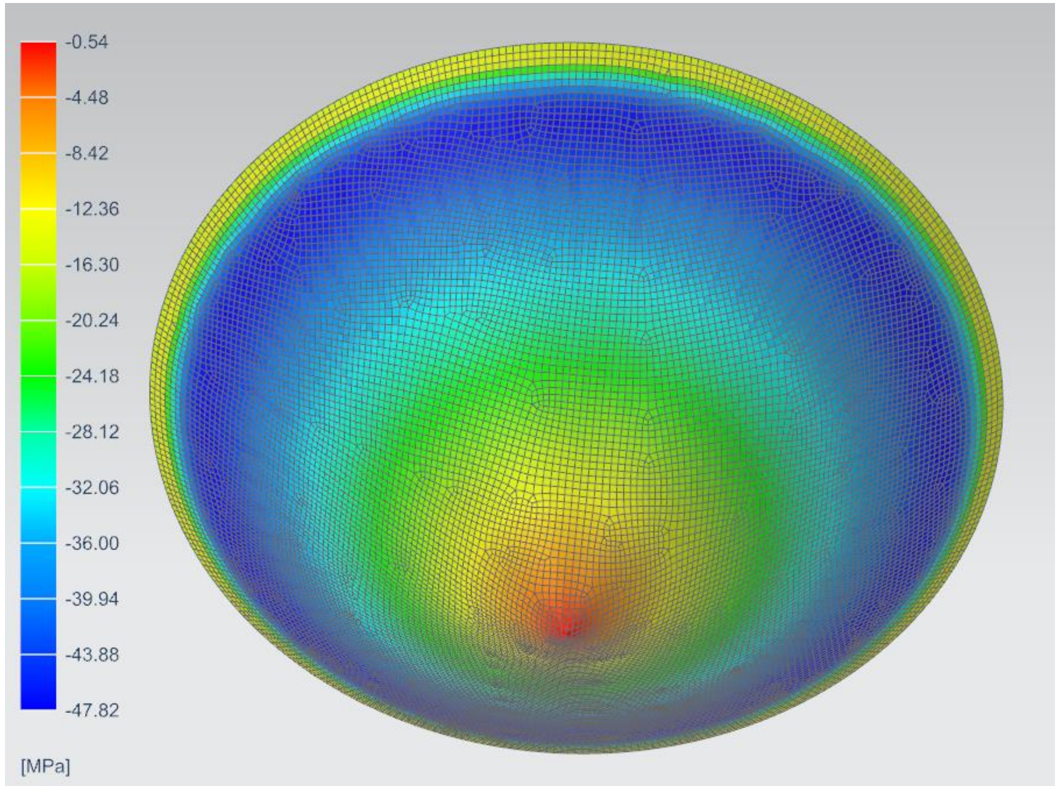
DRAPING – CONIC SHELL EXAMPLE

- Longitudinal Stress

One Dart



Separate Quarters



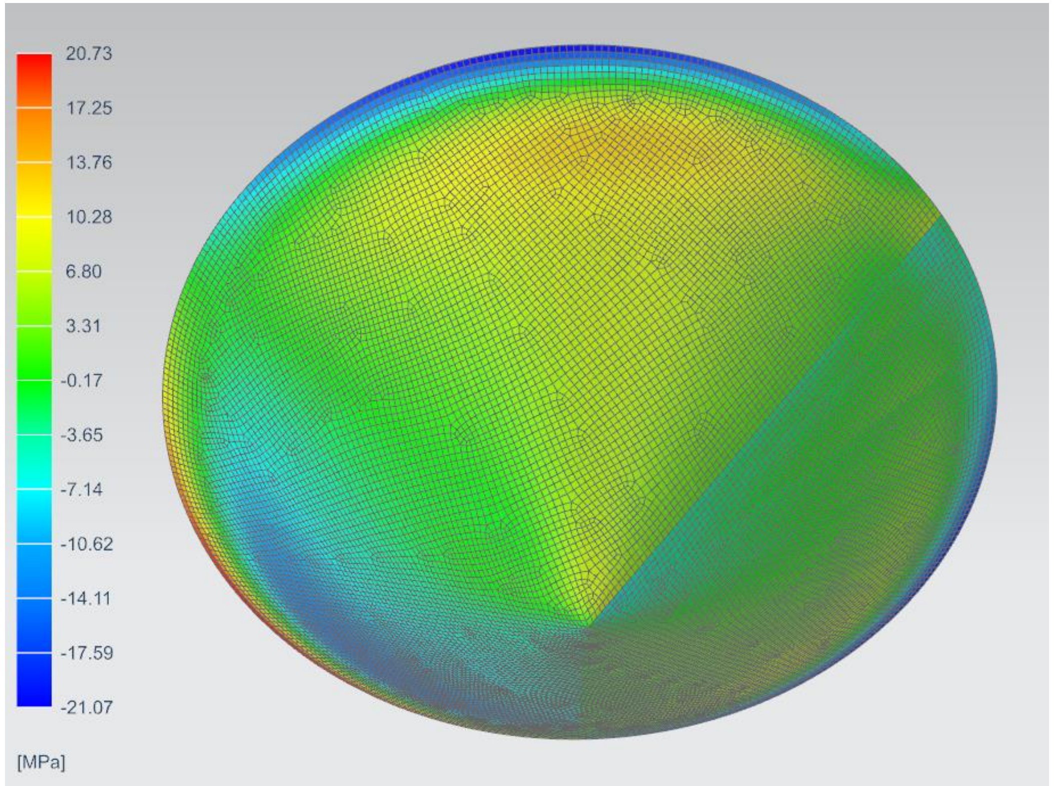
HOSTED BY: CKN Composites Knowledge Network

HOSTED BY: sampe Canada

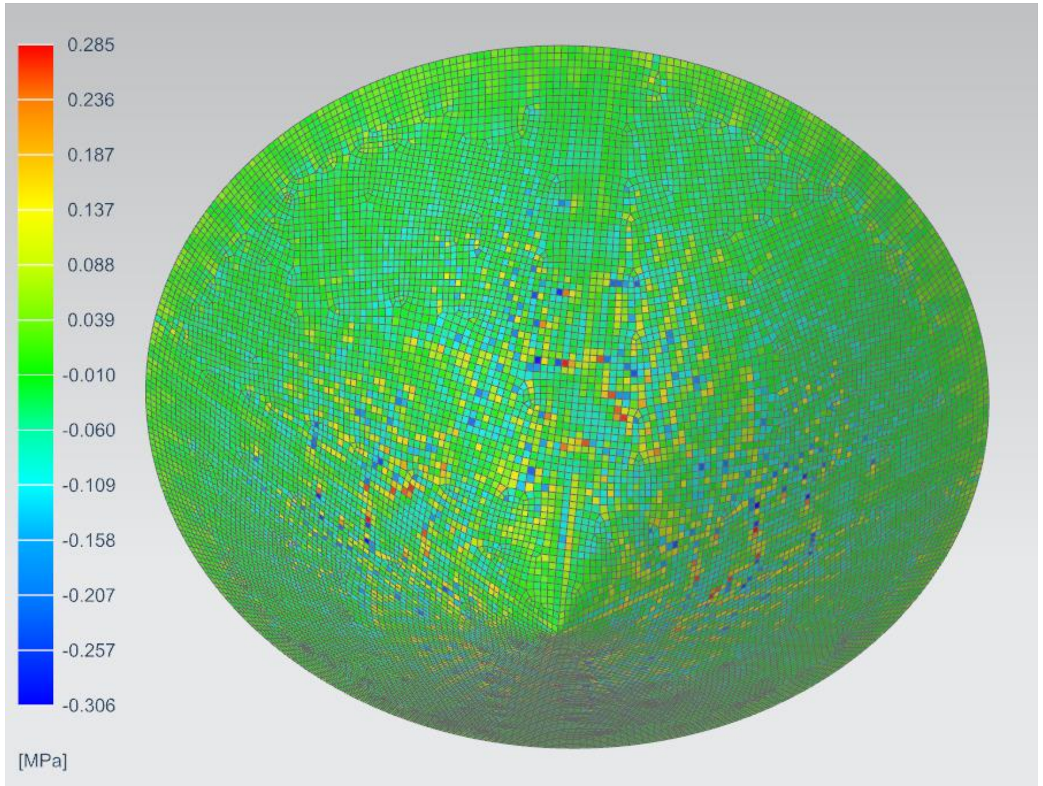
DRAPING – CONIC SHELL EXAMPLE

- Shear Stress

One Dart



Separate Quarters



FAILURE THEORIES

- First Ply Failure
  - Most common approach is to assume that as soon as one ply fails the structures has failed
- Many failure theories exist

X, Y, S	Ply longitudinal, transverse, and shear strengths (subscripts T for tensile and C for compressive)
σ_1, σ_2, τ_{12}	Applied longitudinal, transverse, and shear stresses
ε_1, ε_2, γ_{12}	Applied longitudinal, transverse, and shear strains
E_1, E_2, G_{12}	Ply axial Young's moduli

FAILURE THEORIES

- Maximum stress / strain
  - Do not consider interaction between different types of stresses/strains

$$\frac{\sigma_1}{X} < 1, \qquad \frac{\sigma_2}{Y} < 1, \qquad \frac{\tau_{12}}{S} < 1$$

$$\varepsilon_1 < \frac{X}{E_1}, \qquad \varepsilon_2 < \frac{Y}{E_2}, \qquad \gamma_{12} < \frac{S}{G_{12}}$$

FAILURE THEORIES

- Tsai-Wu
  - Quadratic equation that considers the interaction between different types of stresses, including both tensile and compressive stresses
  - Requires an experimentally derived interaction coefficient, F_{12}
  - Due to practical difficulties in experimentally obtaining this interaction coefficient, it has been suggested that for highly orthotropic materials, F_{12} may be set to zero

$$F_1\sigma_1 + F_{11}\sigma_1^2 + F_2\sigma_2 + F_{22}\sigma_2^2 + 2F_{12}\sigma_1\sigma_2 + F_{66}\tau_{12}^2 < 1$$

$$F_1 = \frac{1}{X_T} - \frac{1}{X_C} \qquad F_{11} = \frac{1}{X_T X_C}$$

$$F_2 = \frac{1}{Y_T} - \frac{1}{Y_C} \qquad F_{22} = \frac{1}{Y_T Y_C}$$

$$F_{66} = \frac{1}{S^2}$$

FAILURE THEORIES

- Hoffman
  - Quadratic equation that takes into account the interaction of shear stresses and both tensile and compressive stresses in the longitudinal and transverse directions.
  - Similar to Tsai-Wu, but makes an explicit assumption regarding the interaction coefficient, F_{12}

$$\frac{{\sigma_1}^2}{X_T X_C} - \frac{\sigma_1 \sigma_2}{X_T X_C} + \frac{{\sigma_2}^2}{Y_T Y_C} - \frac{(X_T - X_C)}{X_C X_T}\sigma_1 - \frac{(Y_T - Y_C)}{Y_C Y_T}\sigma_2 + \left(\frac{\tau_{12}}{S}\right)^2 < 1$$

FAILURE THEORIES

- Hill
  - Quadratic equation that considers the interaction between different types of stresses, but assumes the same strength allowable in tension and compression (i.e. $\sigma_1 = \sigma_t = \sigma_c$)

$$\left(\frac{\sigma_1}{X}\right)^2 - \left(\frac{\sigma_1\sigma_2}{X^2}\right) + \left(\frac{\sigma_2}{Y}\right)^2 + \left(\frac{\tau_{12}}{S}\right)^2 < 1$$

$$X = X_T \text{ if } \sigma_1 > 0$$

$$X = X_C \text{ if } \sigma_1 < 0$$

$$Y = Y_T \text{ if } \sigma_2 > 0$$

$$Y = Y_C \text{ if } \sigma_2 < 0$$

For $\frac{\sigma_1\sigma_2}{X^2}$ term $X = X_T$ if σ_1 and σ_2 are the same sign

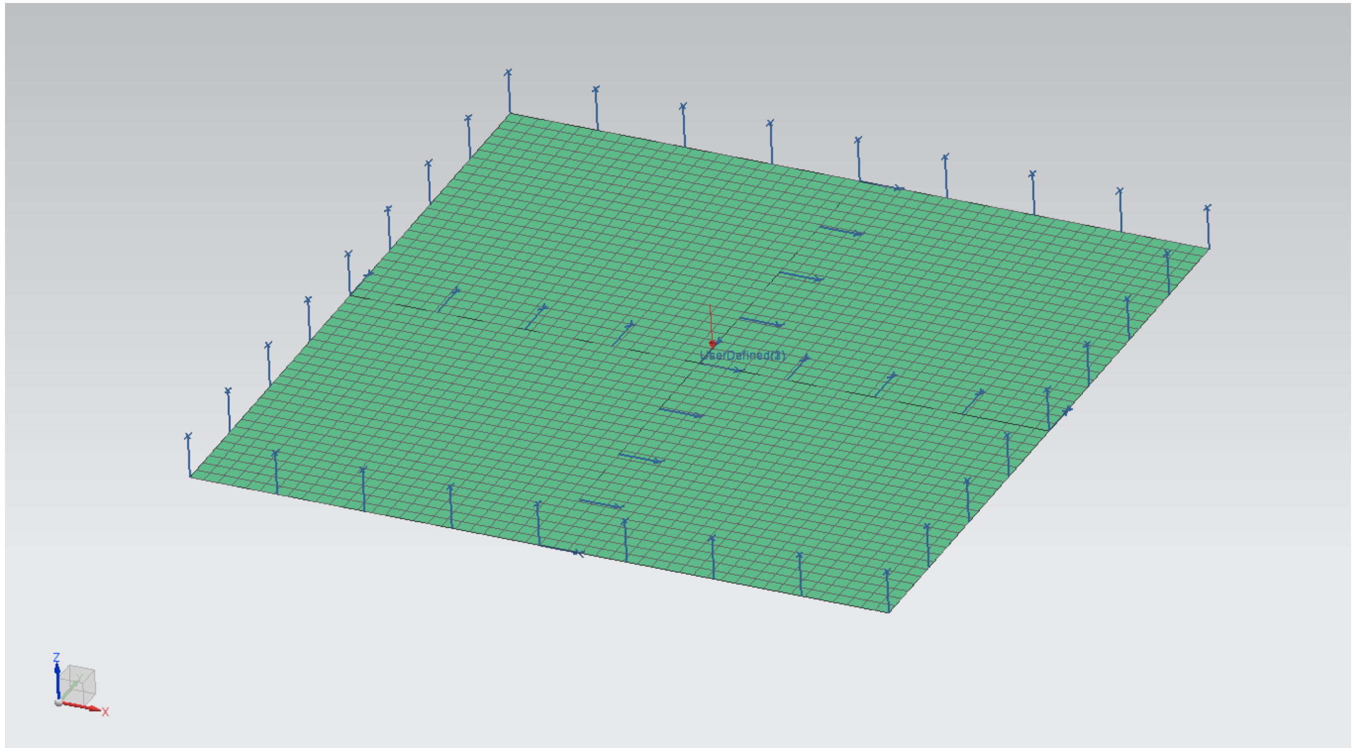
X = X_C if σ_1 and σ_2 are different signs

HOSTED BY: CKN Composites Knowledge Network

HOSTED BY: sampe Canada

FAILURE THEORIES - EXAMPLE

- Square plate 200mm x 200mm
- Eight fibreglass plies with a laminate schedule of [0/90/0/90]s
- Simply supported along all four of its outer edges
- 2000N bending load is applied in the centre of the plate



FAILURE THEORIES - EXAMPLE

Strength Allowables

Longitudinal Tensile Strength (MPa)	276
Transverse Tensile Strength (MPa)	317
Longitudinal Compressive Strength (MPa)	317
Transverse Compressive Strength (MPa)	421
Shear Strength (MPa)	103

Stresses in Top and Bottom Ply

	Ply 1	Ply 8
Longitudinal Stress (MPa)	133.1	-133.1
Transverse Stress (MPa)	132.9	-132.9
Shear Stress (MPa)	4.6	4.6

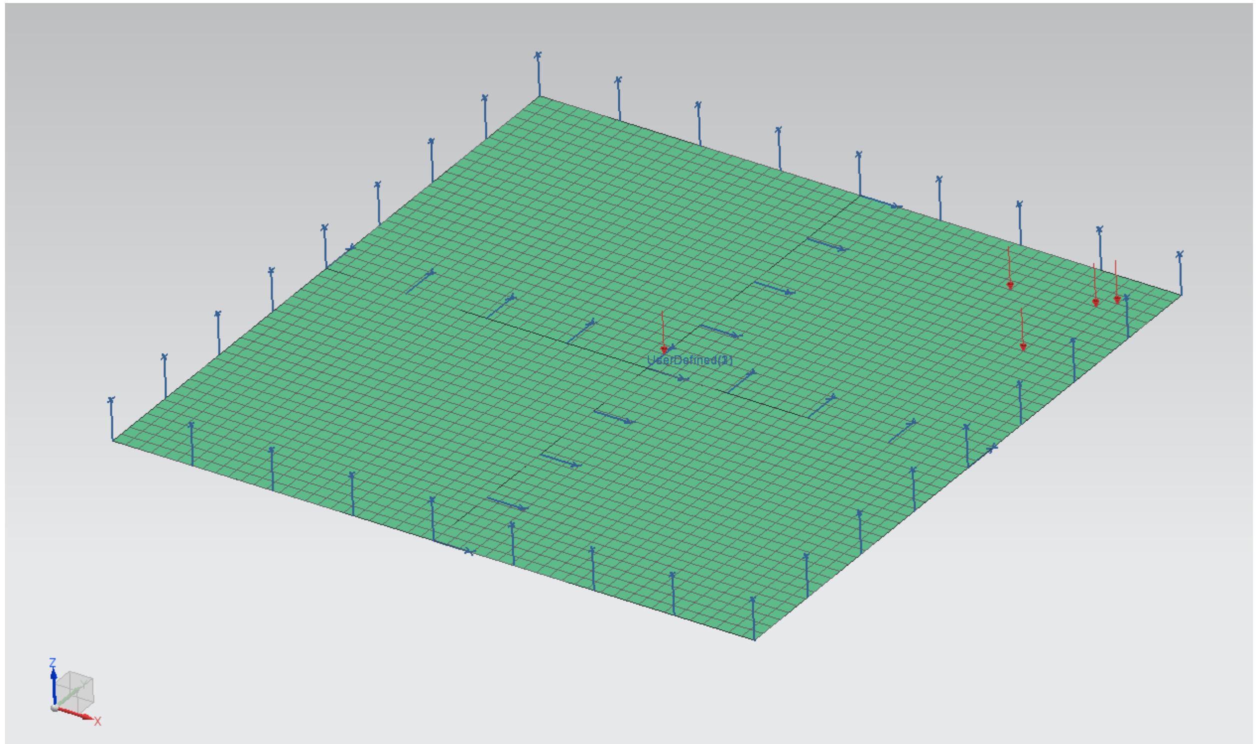
Strength Ratio Results

	Ply 1	Ply 8
Maximum Stress	2.07	2.38
Hill	2.37	4.69
Hoffman	2.18	3.41
Tsai-Wu	1.97	2.93

- Strength ratio higher for ply 8 due to higher compression allowable
- Ply 1 results clustered close together
- Ply 8 shows large differences especially for Hill

FAILURE THEORIES - EXAMPLE

- Example
  - The same analysis setup was employed, but additional forces were applied near the corner to induce nearby shear stresses



FAILURE THEORIES - EXAMPLE

Stresses in Top Ply

	Ply 1
Longitudinal Stress (MPa)	44.4
Transverse Stress (MPa)	43.7
Shear Stress (MPa)	29.1

Strength Ratio Results

	Ply 1
Maximum Stress	3.55
Hill	3.18
Hoffman	2.98
Tsai-Wu	2.92

- Shear stress is now significant
- Max stress considers each stress independently so produces higher strength ratio
- Other theories consider interaction and produce results more closely clustered

INTERPRETING RESULTS

- Determine appropriate strength ratio (safety factor)
- Review overall failure indices or strength ratios
- If failure occurs determine which plies are failing
- Evaluate longitudinal, transverse, and shear stresses in the critical plies
- Assess ply coupling effects if relevant
- Redesign could include:
  - Adding more plies
  - Changing the fibre orientation of individual plies
  - Reordering plies in the laminate stack
  - Adjusting the overall material orientation
  - Making changes to the geometry

Thank you for joining us!

Keep an eye out for upcoming AIM events:

Introduction to Out of Autoclave Prepreg Processing
Hosted by Dr. Casey Keulen
July 30, 2025
https://compositeskn.org/KPC/A393

And don't forget to visit the KPC for more information:

https://compositeskn.org/KPC

Today's Webinar will be posted at:
https://compositeskn.org/KPC/A392

HOSTED BY: CKN Composites Knowledge Network

HOSTED BY: sampe Canada

QUESTIONS

Alastair Komus

alastair@ascomposites.ca

204-333-6773

https://www.ascomposites.ca/

A&S Composites Engineering