A 12 PART WEBINAR SERIES ON:

COMPOSITE MATERIALS ENGINEERING

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YOUR HOST

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- Ph.D. and M.A.Sc. in Composite Materials Engineering
- Over 15 years experience in industry and academia working on polymer matrix composites in aerospace, automotive, marine, energy, recreation and others
- Experience working with over 150 companies from SME to major international corporations
- Expertise in liquid composite moulding and thermal management

PAST WEBINAR RECORDINGS NOW POSTED

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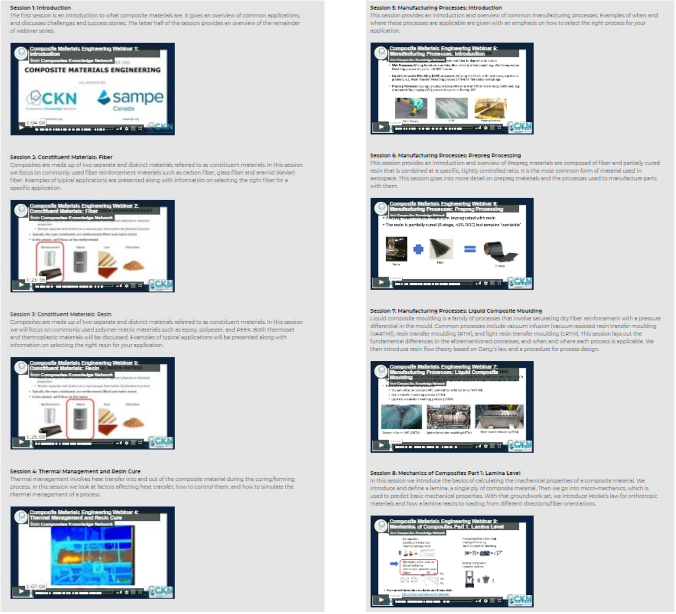

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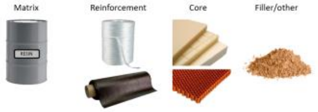
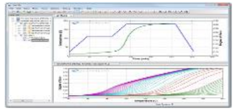
OVERVIEW OF WEBINAR SERIES

- **Series of 12 webinars, 1 hour each**

Introduction
Constituent Materials
Thermal Management

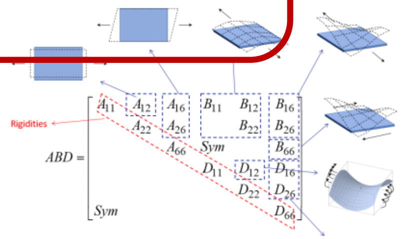




Processing (Manufacturing)
Prepreg Processing
Liquid Composite Moulding

Mechanics of Composites
Micromechanics
Lamina and Laminate Level
Failure

$$\begin{Bmatrix} N \\ M \end{Bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{Bmatrix} \varepsilon_0 \\ \kappa \end{Bmatrix}$$



Testing Composites
Common Defects

- **For more information on dates and times visit:**
  https://compositeskn.org/aimevents/

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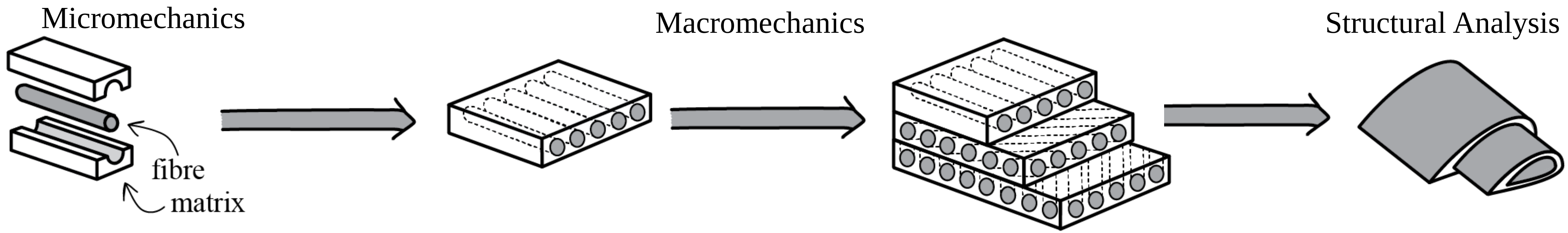
ANALYSIS OF COMPOSITE MATERIALS

The complicated nature of a structure made from composite materials lends to many layers of analysis



- Micro-mechanics focuses on the stresses, strains, damage, etc., happening at the level of individual fibres and the fibre/matrix interface
- Macro-mechanics focuses on the analysis at the level of individual lamina and sub-laminates
- Structural mechanics involves analysis at the structural scale

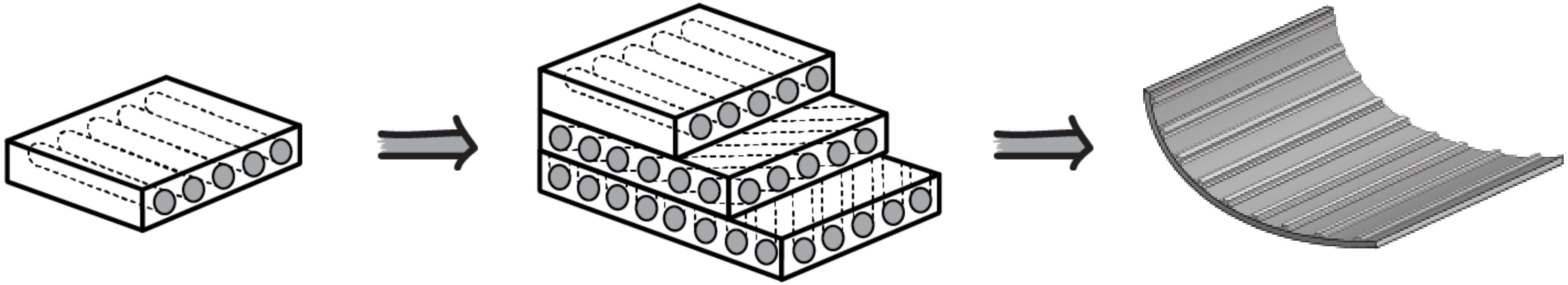
MICROMECHANICS AND MACROMECHAINCS

- Micromechanics:
  - Starting from constituent material properties

- Macromechanics:
  - Starting from the combined properties of a ply of fibre and resin

LAMINATED COMPOSITES

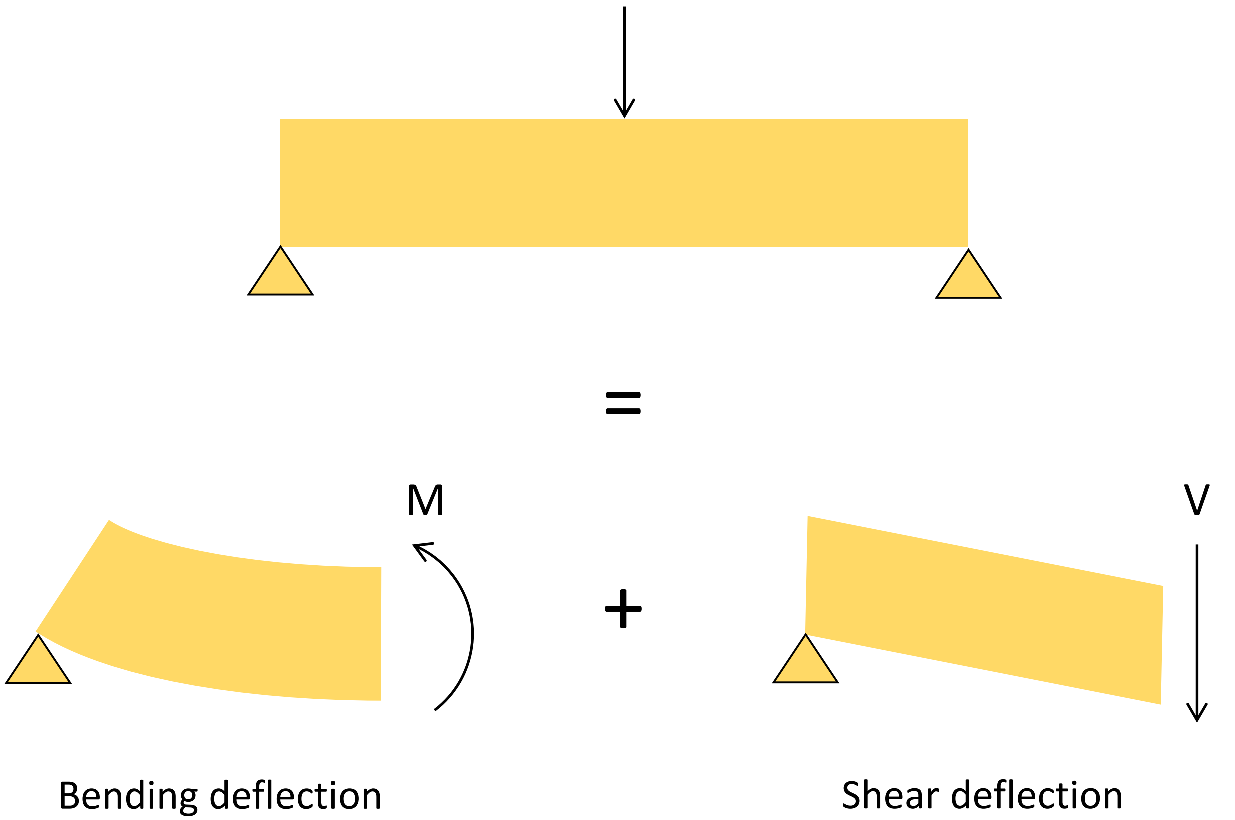
- Laminated composites are made by stacking layers of material (lamina) on top of each other to create a laminate
- Layers are arranged at various orientations (angles) to provide stiffness and strength in desired directions.



Lamina Laminate Component

BEAM THEORIES

- Total deflection in a beam is due to both bending and shear deflections:



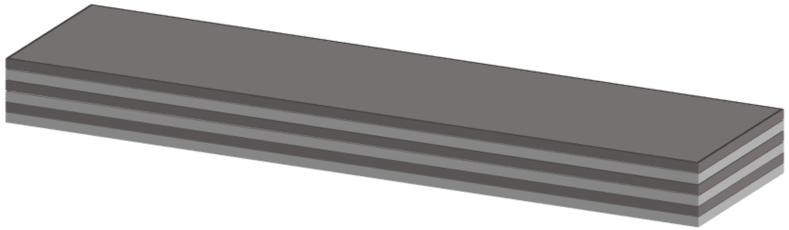
$$\delta = \delta_{bending} + \delta_{shear}$$

BEAM THEORIES

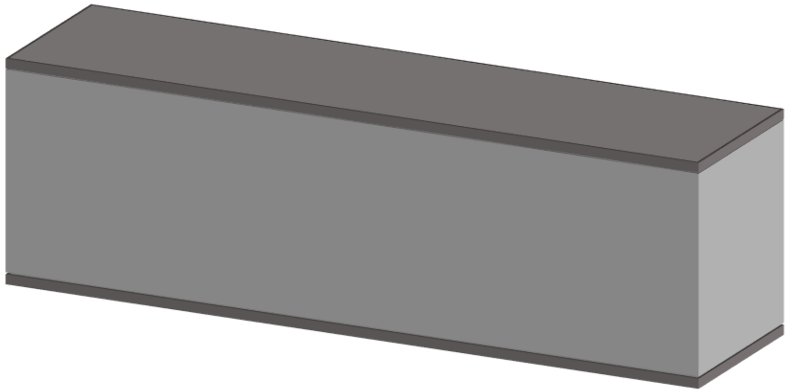
- There are two common beam bending theories:
  - Euler-Bernoulli beam theory: neglects the shear deflection
  - Timoshenko beam theory: considers both shear and bending deflections
- In most cases, shear deflection is much smaller than the bending deflection and can be neglected:
  - Thin beams/plates where the span/thickness ratio is larger than 10
- In thick beams and sandwich panels, the shear deflection becomes large and cannot be neglected

LAMINATION THEORIES

Classical Lamination Theory (CLT)	Sandwich Theory
Ignoring the shear deflection (based on *Euler-Bernoulli* Theory)	Including the shear deflection (based on *Timoshenko* Theory)



For thin laminates only

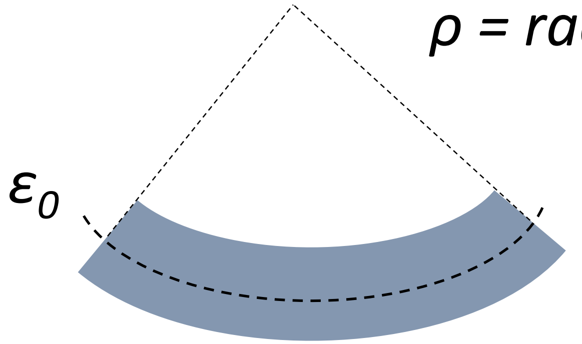


For sandwich panels with thin facesheets and thick cores

CLASSICAL LAMINATION THEORY: LAMINATED BEAM

- Two types of loading are considered for a laminated beam: axial load and bending moment:

- Two types of deformation are considered for a laminated beam: mid-plane axial strain and curvature:

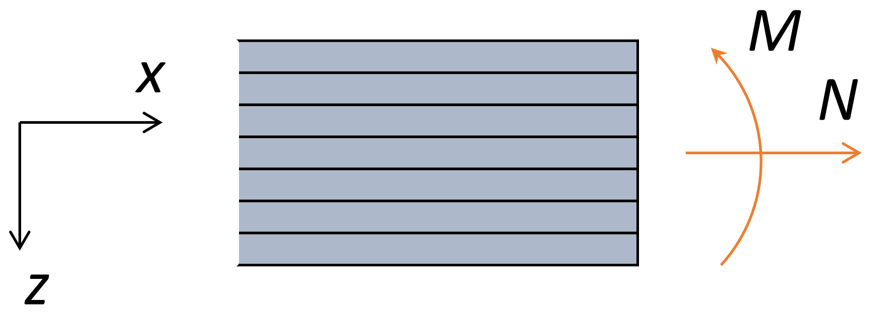


$\rho = radius, \ \ \kappa = 1/\rho$

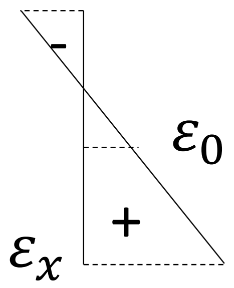
Curvature: Kappa

CLASSICAL LAMINATION THEORY: LAMINATED BEAM

- Strains through the cross section of the beam are calculated from:



Laminated Beam Cross Section

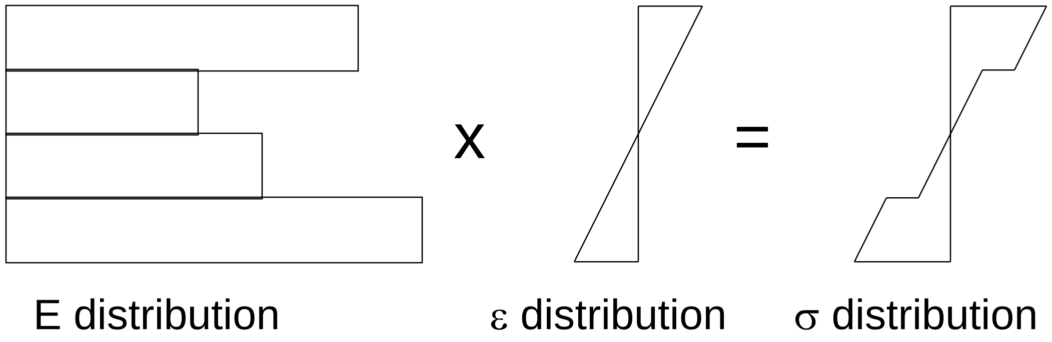


$$\varepsilon_x = \varepsilon_0 + z\kappa$$

- ε_0 is the strain at the centre point of the beam
- κ is the curvature of the beam
- z is the location of the ply

CLASSICAL LAMINATEION THEORY: LAMINATED BEAM

- For layered and non-homogeneous beams, the stress distribution through the beam thickness is not smooth:



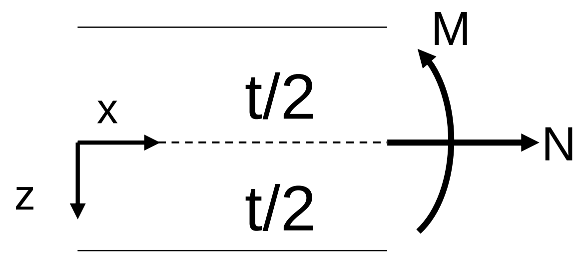
- Note that the linear strain distribution relies on the assumption that there is perfect bonding between the layers

STRESS ANALYSIS

- Consider a section of the beam. The stresses in the section lead to an equivalent axial force, *N*, and a bending moment, *M:*

$$N = \int_{\frac{-t}{2}}^{\frac{t}{2}} \sigma_x dz = \text{axial load / unit width}$$

$$M = \int_{\frac{-t}{2}}^{\frac{t}{2}} \sigma_x \cdot z dz = \text{moment / unit width}$$



- Substituting for σ_x,

$$N = \int_{-t/2}^{t/2} \sigma_x dz = \int_{-t/2}^{t/2} E\varepsilon_x dz = \int_{-t/2}^{t/2} E(\varepsilon_0 + z\kappa) dz$$

STRESS ANALYSIS

- Therefore:

$$N = \left(\int_{-t/2}^{t/2} E dz \right) \varepsilon_0 + \left(\int_{-t/2}^{t/2} E z dz \right) \kappa$$

$$N = A\varepsilon_0 + B\kappa$$

- Similarly for moment:

$$M = \left(\int_{-t/2}^{t/2} E z dz \right) \varepsilon_0 + \left(\int_{-t/2}^{t/2} E z^2 dz \right) \kappa$$

$$M = B\varepsilon_0 + D\kappa$$

STRESS ANALYSIS

- Or, if we write the equations in matrix form:

$$\begin{Bmatrix} N \\ M \end{Bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{Bmatrix} \varepsilon_0 \\ \kappa \end{Bmatrix}$$

  - N = Axial force/beam width
  - M = Moment/beam width
  - ε_0 = Strain at the centre line of the beam
  - κ = beam curvature

STRESS ANALYSIS

- The components of the matrix are:

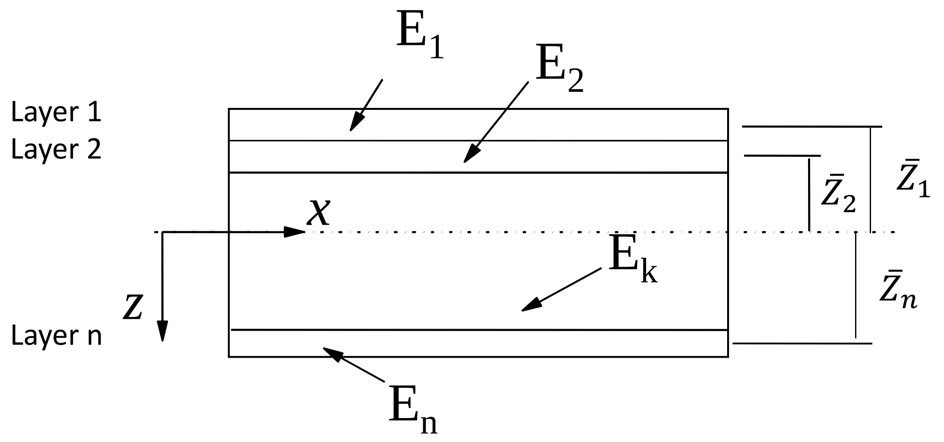
$$A = \int_{\frac{-t}{2}}^{\frac{t}{2}} E dz = \text{extensional stiffness of the beam}$$

$$B = \int_{\frac{-t}{2}}^{\frac{t}{2}} E \cdot z \cdot dz = \text{coupling stiffness of the beam}$$

$$D = \int_{\frac{-t}{2}}^{\frac{t}{2}} E \cdot z^2 \cdot dz = \text{bending stiffness of the beam}$$

LAMINATED BEAM

$\bar{Z}_i$: Distance between center of each ply and center of the beam
E_i: Modulus of each lamina along direction x
t_i: Thickness of each lamina



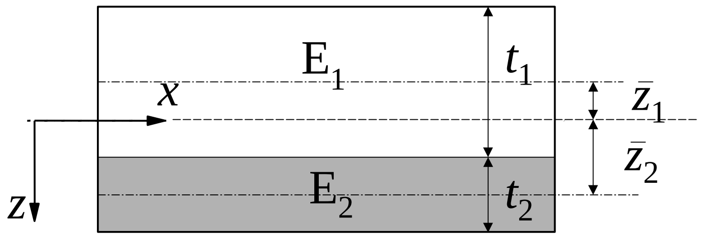
$$A = \sum_{k=1}^{n} E_k \cdot t_k$$ > 0

$$B = \sum_{k=1}^{n} E_k \cdot t_k \cdot \bar{z}_k$$ = 0 for symmetric and homogeneous laminates

$$D = \sum_{k=1}^{n} E_k \cdot \left(t_k \bar{z}_k^2 + \frac{t_k^3}{12} \right)$$ > 0

LAMINATED BEAM EXAMPLE

- Consider a 2 layered beam:
- Find the stiffness matrix



$$A = \sum_{k=1}^{n} E_k \cdot t_k$$

$$A = E_1 \cdot t_1 + E_2 \cdot t_2$$

$$\bar{z}_1 = -\left(\frac{t_1 + t_2}{2} - \frac{t_1}{2} \right) = -\frac{t_2}{2}$$

$$\bar{z}_2 = \left(\frac{t_1 + t_2}{2} - \frac{t_2}{2} \right) = \frac{t_1}{2}$$

LAMINATED BEAM EXAMPLE

$$B = \sum_{k=1}^{2} E_k \cdot t_k \cdot \bar{z}_k = E_1 t_1 \bar{z}_1 + E_2 t_2 \bar{z}_2$$

$$B = E_1 t_1 \left(\frac{-t_2}{2} \right) + E_2 t_2 \left(\frac{t_1}{2} \right)$$

$$B = \frac{t_1 t_2}{2} \left(E_2 - E_1 \right)$$

For homogeneous beams, E_1=E_2 so B=0 (i.e. no axial bending coupling)

$$D = \sum_{k=1}^{n} E_k \cdot \left(t_k \bar{z}_k^2 + \frac{t_k^3}{12} \right)$$

$$D = E_1 \left(\frac{t_1 t_2^2}{4} + \frac{t_1^3}{12} \right) + E_2 \left(\frac{t_2 t_1^2}{4} + \frac{t_2^3}{12} \right)$$

LAMINATE CODE

- A laminate is made from stacked laminae
- Each layer is identified by its location, material, and angle with respect to some reference direction
- Each lamina is represented by the angle of the ply and is separated from other plies by a '/'
- The first ply is the top ply
- For example:

0°
-45°
90°
60°
30°

= [0/-45/90/60/30]

This implies that there are 5 plies, each of which has a different angle to the reference direction (0°), and each ply is the same material, and the same thickness.

LAMINATE CODE

Example:

0°
-45°
90°
90°
60°
0°

= $[0/-45/90_2/60/0]$

This implies that there are 6 plies, all of the same material, with two 90° plies in the centre of the laminate. A subscript within the laminate code indicates additional plies.

SYMMETRIC LAMINATES

0°
-45°
60°
60°
-45°
0°

$= [0/-45/60]_S$

This implies that there are 6 plies, all of the same material, with a symmetric layup. The centre plane of the laminate mirrors the plies exactly.

With symmetric laminates,

$$\overline{Q}(z) = \overline{Q}(-z) \rightarrow [B] = [0]$$

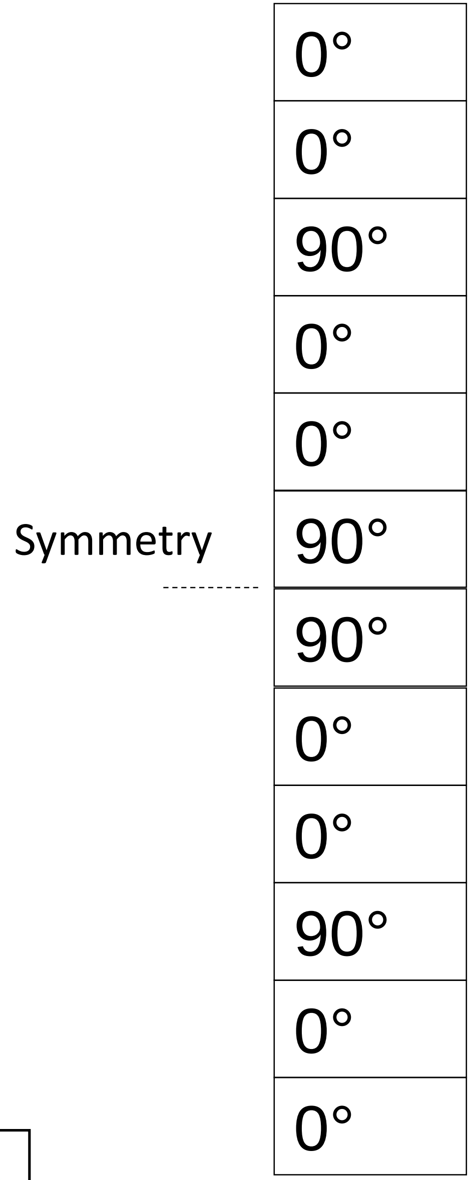
$$\begin{Bmatrix} \{N\} \\ \{M\} \end{Bmatrix} = \begin{bmatrix} [A]_{3\times3} & [0]_{3\times3} \\ [0]_{3\times3} & [D]_{3\times3} \end{bmatrix} \begin{Bmatrix} \{\varepsilon^O\} \\ \{\kappa\} \end{Bmatrix}$$

0°
-45°
60°
-45°
0°

$= [0/-45/\overline{60}]_S$

This implies that there are 5 plies, all of the same material, with a symmetric layup. The centre ply is the 60° ply, indicated by the over-bar, and is not repeated beneath the symmetry line (since the line of symmetry is in the middle of the ply).

LAMINATE CODE



$= [0_2/90]_{2S}$

This implies that there are 12 plies, all of the same material. A subscript within the laminate code indicates additional plies. A subscript outside the laminate code indicates repeats of the same code.

QUASI-ISOTROPIC LAMINATES

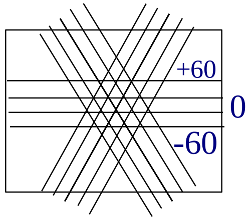
- When 3 or more identical plies are stacked at equal angles (ie the angle separating each of n plies is π/n) then the resulting *in-plane* properties are isotropic

0°
60°
-60°

$= [0/60/120]_T$
$n=3;\ \pi/3=60°$

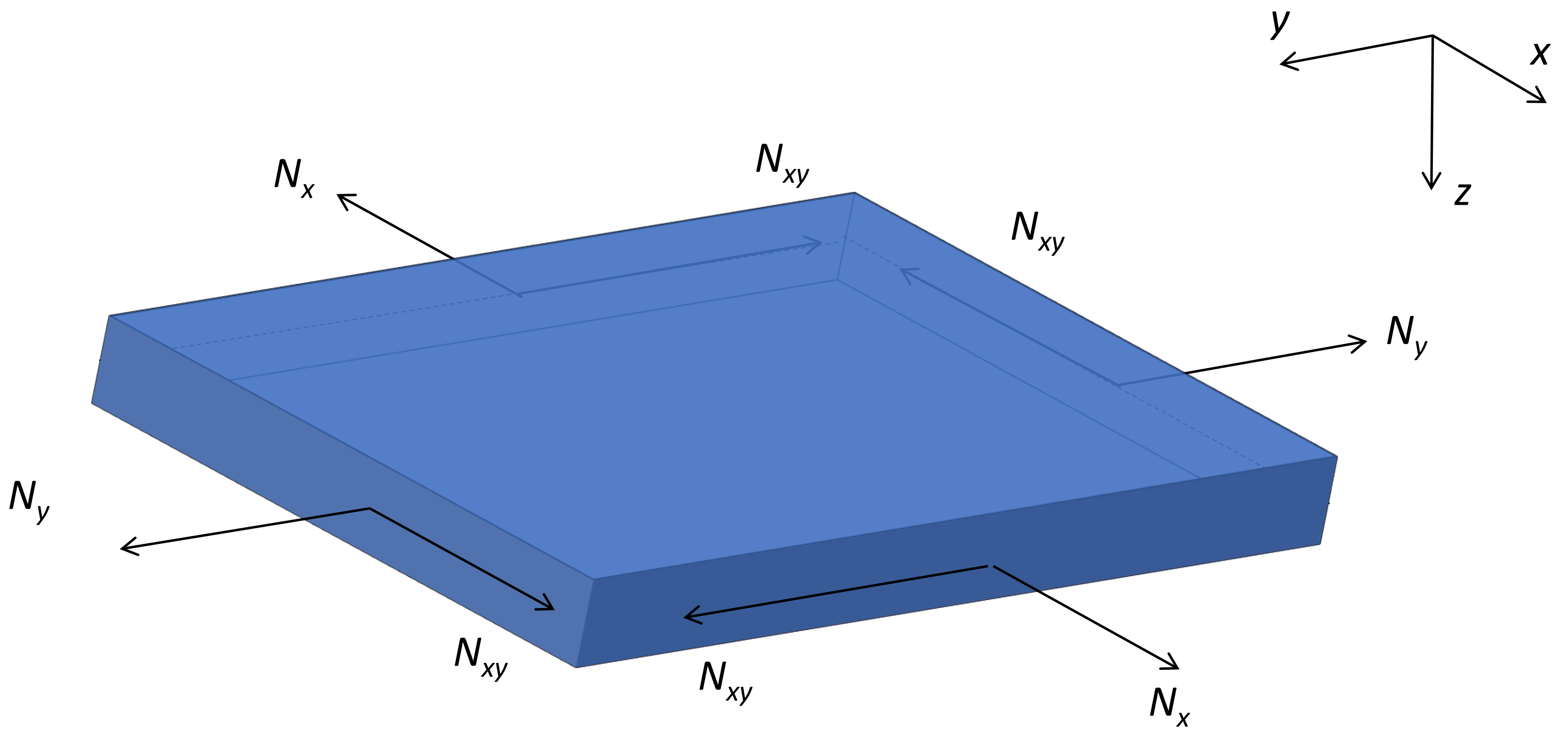
0°
45°
90°
-45°

$= [0/45/90/-45]_T$
$n=4;\ \pi/4=45°$



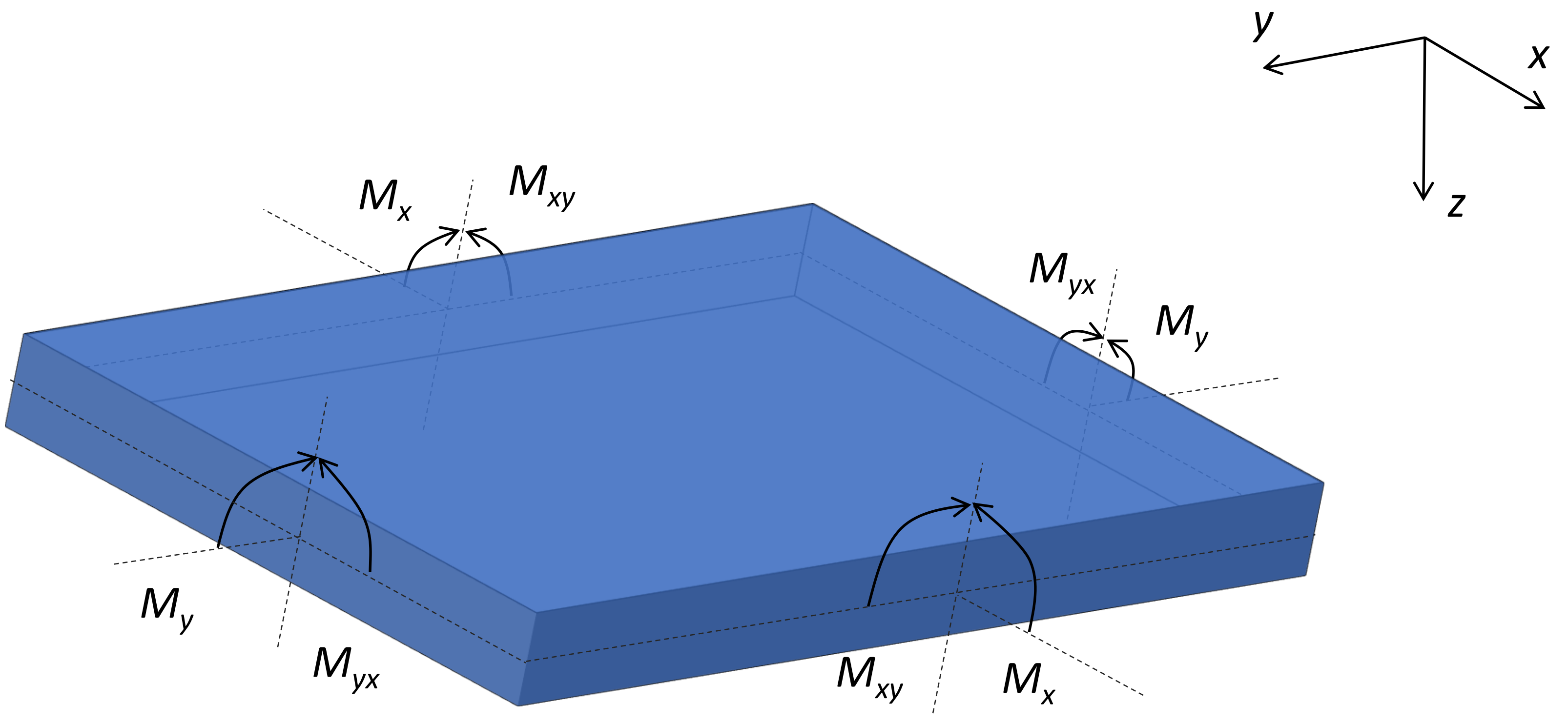
For a quasi-isotropic laminate, it can be shown that the in-plane stiffness coefficients A_{ij}=constant. That is, at any angle, the in-plane stiffness is the same. This is quasi-isotropic since the bending stiffnesses (D_{ij}) are not constant.

LAMINATED PLATE - FORCES



Through thickness forces (N_z, N_{zx} and N_{yz}) are ignored for a thin plate

LAMINATED PLATE - MOMENTS



Through thickness moments (Mz, Mzx and Myz) are ignored for a thin plate

LAMINATED PLATE ANALYSIS

- The stiffness matrix for a laminated plate becomes:

$$\begin{Bmatrix} \{N\} \\ \{M\} \end{Bmatrix} = \begin{bmatrix} [A]_{3\times3} & [B]_{3\times3} \\ [B]_{3\times3} & [D]_{3\times3} \end{bmatrix} \begin{Bmatrix} \{\varepsilon^O\} \\ \{\kappa\} \end{Bmatrix}$$

- Or,

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ & A_{22} & A_{26} & & B_{22} & B_{26} \\ Sym & & A_{66} & Sym & & B_{66} \\ & & & D_{11} & D_{12} & D_{16} \\ & & & & D_{22} & D_{26} \\ Sym & & & Sym & & D_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^O \\ \varepsilon_y^O \\ \gamma_{xy}^O \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}$$

LAMINATED PLATE ANALYSIS

- The stiffness matrix components can be written as:

$$[A] = \sum_{k=1}^{n} [\overline{Q}]_k \cdot t_k \qquad A_{11} = \sum_{k=1}^{n} (\overline{Q}_{11})_k \cdot t_k$$

$$[B] = \sum_{k=1}^{n} [\overline{Q}]_k \cdot t_k \cdot \overline{z}_k \qquad B_{11} = \sum_{k=1}^{n} (\overline{Q}_{11})_k \cdot t_k \cdot \overline{z}_k$$

$$[D] = \sum_{k=1}^{n} [\overline{Q}]_k \cdot \left(t_k \overline{z}_k^2 + \frac{t_k^3}{12} \right) \qquad D_{11} = \sum_{k=1}^{n} (\overline{Q}_{11})_k \cdot \left(t_k \overline{z}_k^2 + \frac{t_k^3}{12} \right)$$

- Where $\overline{Q}$ is the transformed stiffness matrix of each layer.

COMPONENTS OF ABD MATRIX FOR ISOTROPIC MATERIAL

- ABD matrix for a homogenous plate made of an isotopic material (e.g. steel):

ABD matrix (isotropic material)

$$\begin{bmatrix} \frac{Et}{1-\nu^2} & \frac{\nu Et}{1-\nu^2} & 0 & 0 & 0 & 0 \\ & \frac{Et}{1-\nu^2} & 0 & 0 & 0 & 0 \\ & & Gt & 0 & 0 & 0 \\ & & & \frac{Et^3/12}{1-\nu^2} & \frac{\nu Et^3/12}{1-\nu^2} & 0 \\ & & & & \frac{Et^3/12}{1-\nu^2} & 0 \\ Sym & & & & & Gt^3/12 \end{bmatrix}$$

(ABD)’ = Inverse of ABD

$$\begin{bmatrix} \frac{1}{Et} & \frac{-\nu}{Et} & 0 & 0 & 0 & 0 \\ & \frac{1}{Et} & 0 & 0 & 0 & 0 \\ & & \frac{1}{Gt} & 0 & 0 & 0 \\ & & & \frac{1}{Et^3/12} & \frac{-\nu}{Et^3/12} & 0 \\ & & & & \frac{1}{Et^3/12} & 0 \\ Sym & & & & & \frac{1}{Gt^3/12} \end{bmatrix}$$

COMPONENTS OF ABD MATRIX FOR ISOTROPIC MATERIAL

Coupling of axial strains (ϵ_x and ϵ_y) = Poisson's Effect

Inverse of ABD

$$\begin{bmatrix} \frac{1}{Et} & \frac{-\nu}{Et} & 0 & 0 & 0 & 0 \\ & \frac{1}{Et} & 0 & 0 & 0 & 0 \\ & & \frac{1}{Gt} & 0 & 0 & 0 \\ & & & \frac{1}{Et^3/12} & \frac{-\nu}{Et^3/12} & 0 \\ & & & & \frac{1}{Et^3/12} & 0 \\ Sym & & & & & \frac{1}{Gt^3/12} \end{bmatrix}$$

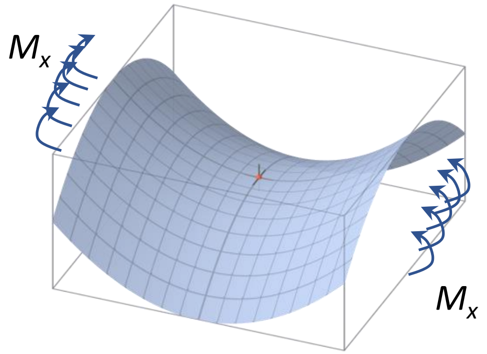
Axial Rigidity

EA

Shear Rigidity

GA

Coupling of curvatures (κ_x and κ_y)



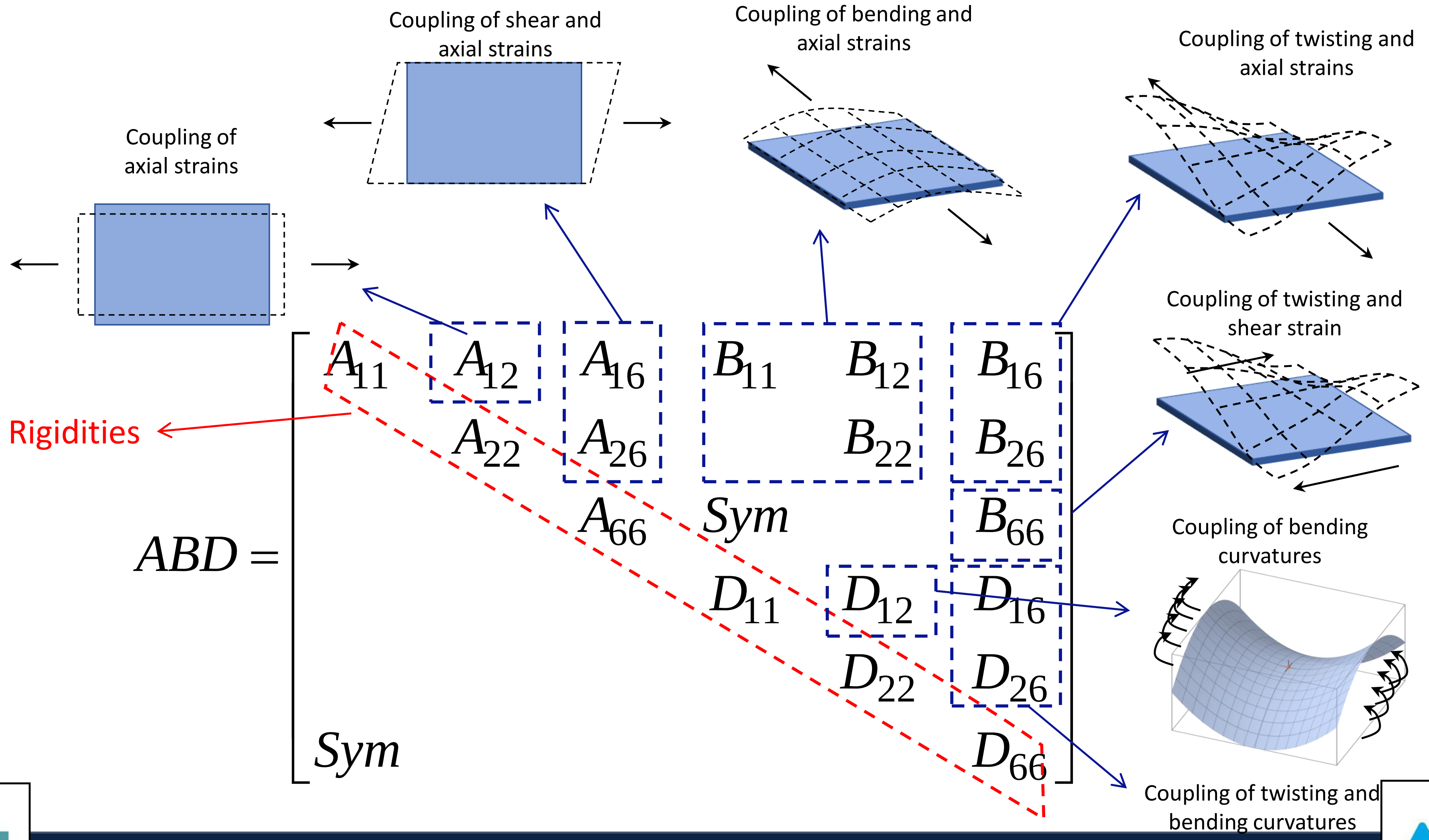
EI

Bending Rigidity

GJ

Torsional Rigidity

COMPONENTS OF ABD MATRIX FOR COMPOSITE LAMINATE

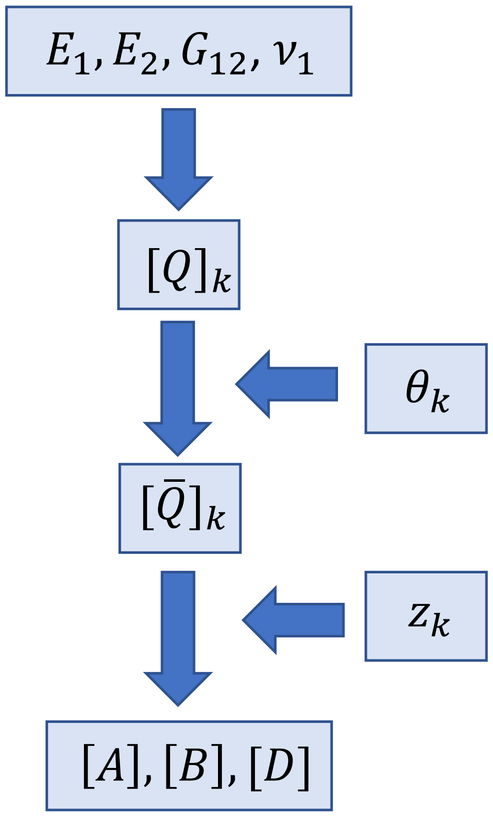


$$ABD = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ & A_{22} & A_{26} & & B_{22} & B_{26} \\ & & A_{66} & Sym & & B_{66} \\ & & & D_{11} & D_{12} & D_{16} \\ & & & & D_{22} & D_{26} \\ Sym & & & & & D_{66} \end{bmatrix}$$

LAMINATED PLATE ANALYSIS

- The [A], [B], and [D] matrices are called the extensional, coupling, and bending stiffness matrices
- The extensional stiffness matrix [A] relates the resultant in-plane forces to the in-plane strains
- The bending stiffness matrix [D] relates the resultant bending moments to the plate curvatures
- The coupling stiffness matrix [B] couples the force and moment terms (an applied axial force will result in a curvature or an applied moment will result in axial extension)

DETERMINING THE STIFFNESS OF A LAMINATE



Elastic properties of each ply

Stiffness of each ply in 1-2 coordinates

Orient each ply

Stiffness of each ply in x-y coordinates

Location of each ply in laminate

Laminate stiffness properties

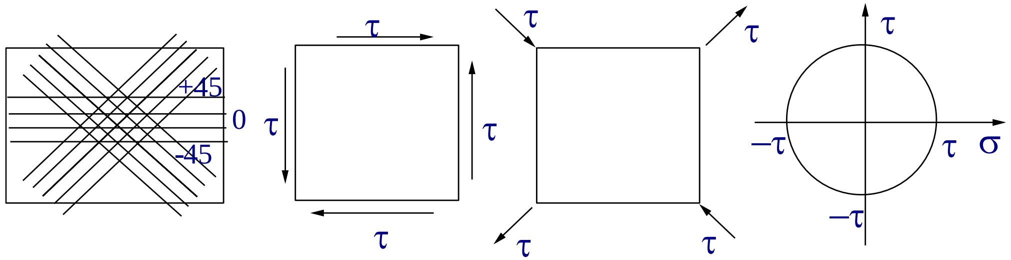
GENERAL LAMINATE CONSIDERATIONS

- For bending applications, it is best to put the plies with the largest axial stiffness (ie 0° plies) as far away from the mid-plane as possible.

$$[D] = \sum_{k=1}^{n} [\overline{Q}]_k \cdot \left(t_k \bar{z}_k^2 + \frac{t_k^3}{12} \right)$$

0
90
90
0

- For torsion and shear applications, include 45° plies



STEPS TO STRESS ANALYSIS IN LAMINATES

For a laminated composite subjected to applied forces and moments:

1. Find the stiffness matrix [Q] for each material
2. Find the transformed stiffness matrix $[\bar{Q}]$ for each layer
3. Find the location of the top of the laminate (z_0) and the location of the bottom of each layer (z_k) with respect to the centre (mid-plane) of the laminate
4. Using the $[\bar{Q}]$ matrices, find [A], [B], [D]
5. If the loads are given, calculate the mid-plane strains and curvatures

$$\begin{Bmatrix} \{\varepsilon^O\} \\ \{\kappa\} \end{Bmatrix} = \begin{bmatrix} [A]_{3\times3} & [B]_{3\times3} \\ [B]_{3\times3} & [D]_{3\times3} \end{bmatrix}^{-1} \begin{Bmatrix} \{N\} \\ \{M\} \end{Bmatrix}$$

STEPS TO STRESS ANALYSIS IN LAMINATES

6. If the mid-plane strains and curvatures are known, find the applied forces and moments (if needed)

$$\begin{Bmatrix} \{N\} \\ \{M\} \end{Bmatrix} = \begin{bmatrix} [A]_{3\times3} & [B]_{3\times3} \\ [B]_{3\times3} & [D]_{3\times3} \end{bmatrix} \begin{Bmatrix} \{\varepsilon^O\} \\ \{\kappa\} \end{Bmatrix}$$

7. For locations of interest, say at location *l* in layer k, find the strains:

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}_l = \begin{Bmatrix} \varepsilon_x^O \\ \varepsilon_y^O \\ \gamma_{xy}^O \end{Bmatrix} + z_l \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}$$

8. Calculate the stresses at location l:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}_l = \left[\overline{Q}\right]_k \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}_l$$

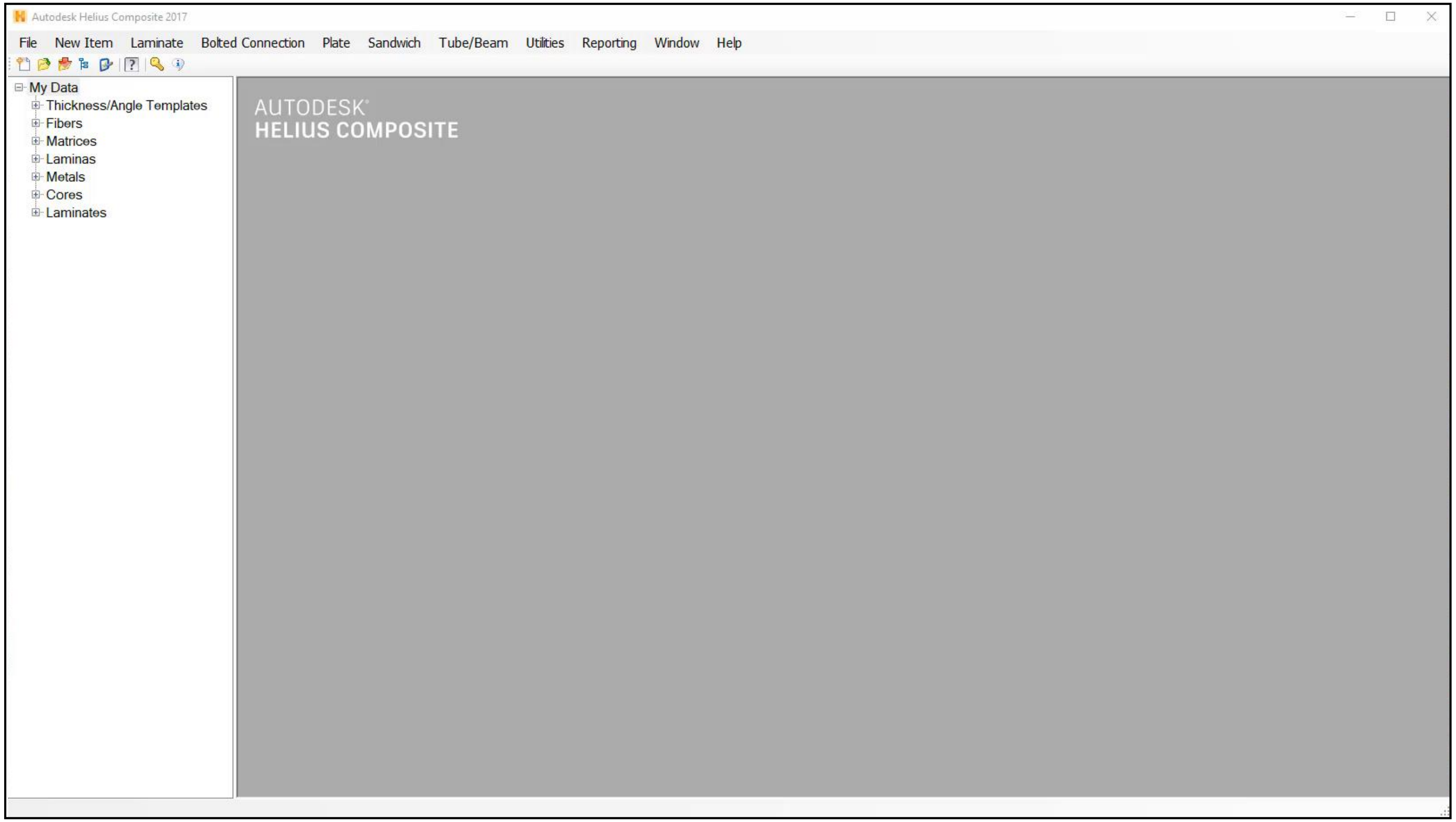
STEPS TO STRESS ANALYSIS IN LAMINATES

9. Calculate the *lamina* (local) stresses or strains:

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix}_l = [T]_k \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}_l \qquad \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix}_l = [T^*]_k \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}_l$$

10. Compare the lamina (local) stresses or strains with the maximum allowables using failure criteria (next session)

COMMERCIAL SOFTWARE



Thank you for joining us!

The next session is:

Session 10: Failure of Composites

October 14, 2020 @ 9:00 am PT

Questions?

For more information on future dates and times visit:

compositeskn.org